Probability Review Assignment
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Please submit this at the start of the first session of “Statistical Foundations” (August 8) during Orientation Week. In each case please show the essential steps of your work. Also, if possible, please do this in the space provided.

(15) 1. Service times at a local fast food restaurant are less than 5 minutes with a probability of 80%. However, when service times are less than 5 minutes, the order has not been correctly filled 10% of the time. When the service time is 5 minutes or longer, the order is correctly filled 95% of the time. What is the overall probability that an order will not be correctly filled?

HINT: Again, it helps to organize the information in a tree! For example, define:

\[ F = \text{event of fast service (less than 5 minutes),} \]
\[ C = \text{event the order is correctly filled.} \]

Then we want to find \( P(C) \) and to do this one can use the following tree.

<table>
<thead>
<tr>
<th>Step 1</th>
<th>Step 2</th>
<th>Outcome</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>( F(\ ) )</td>
<td>( C )</td>
<td>( \bar{C} )</td>
<td>( \cdot )</td>
</tr>
<tr>
<td>( \bar{F}(\ ) )</td>
<td>( C )</td>
<td>( \bar{C} )</td>
<td>( \cdot )</td>
</tr>
</tbody>
</table>

For examples of completed trees, see Lecture 1 of the Probability Review Lectures: http://www2.owen.vanderbilt.edu/bruce.cooil/Probability-Review.pdf.

(15) 2. An investor is considering whether to make a $500,000 investment in an apartment complex. After some research, she estimates the following probability distribution for 4-year net return (in thousands) to the $500,000 investment.

<table>
<thead>
<tr>
<th>4-year net return:</th>
<th>-200</th>
<th>-100</th>
<th>0</th>
<th>100</th>
<th>200</th>
<th>300</th>
<th>400</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability:</td>
<td>0.1</td>
<td>0.1</td>
<td>0.2</td>
<td>0.2</td>
<td>0.1</td>
<td>0.1</td>
<td></td>
</tr>
</tbody>
</table>

a. What is the expected net return (after 4 years)?
b. What is the probability of a nonnegative net return (after 4 years)?

c. What is the standard deviation of 4-year net return?

(20) 3. I have 5 interviews scheduled for positions in which I am interested. Subjectively, I estimate that there is a probability of 0.7 (or 70%) that I will get an offer in each case. Assume that the outcomes for each of the 5 positions are independent of one another. Use my subjective estimate of the probability of success to answer the following questions.

a. Consider the random variable defined as X= number of job offers. What type of probability distribution does X have? (If it’s binomial, it’s enough to say “Binomial” and to specify the values of n and p.)

b. What will be my expected number of job offers?

c. What is the probability that I will get at least 1 offer?

d. What is the probability that I will get at least 3 offers?

(25) 4. A business consultant will work on 10 projects during the next 3 months. There is a 0.4 probability that he will complete any given project, and the projects are independent (e.g., the completion of one project does not affect the probability that another will be completed).

a. Let X represent the number of projects completed during the next 3 months. What kind of probability distribution does X have? (If it’s binomial, be sure to specify n and p.)

b. What is the expected number of projects that will be completed during the next 3 months?
c. What is the probability that he will complete exactly 4 projects during the next 3 months?

d. The consultant's income (in thousands of dollars) is represented by the random variable I, and I = 10 + 20X, where X is the number of projects completed.

[1] What is the expected income during the next 3 months?

[2] What is the standard deviation of income during the next 3 months.

HINT: Part (d) does not require a lot of work. Recall that:
Expected value of I = 10 + 20(Expected Value of X), &
Standard Deviation of I = 20(Standard Deviation of X).

(10) 5. I have invested in a security with an historical annual mean return of 7% and a standard deviation of 1.5%. Define the random variable X: X = % return on security during next year. Assume this random variable has a normal distribution.

a. What is the probability that the return during the next year will be less than 4%?

b. What is the probability the return during the next year will exceed 8.5%?

(40) 6. The following table summarizes the mean annual return and standard deviation (both in %) for mutual funds in two different investment classes.

<table>
<thead>
<tr>
<th>Investment Class</th>
<th>Mean (%)</th>
<th>Standard Deviation (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Domestic Large Caps</td>
<td>11.71</td>
<td>15.30</td>
</tr>
<tr>
<td>Domestic Small Caps</td>
<td>14.93</td>
<td>21.82</td>
</tr>
</tbody>
</table>

Please answer the following questions, assuming the returns in each class are normally distributed and that we are only considering annual returns.

a. For each of the two investment classes above, what is the probability of a loss?
b. For each mutual fund, what is the probability of a return that is
[1] less than 7%?

[2] more than 10%?

c. Without doing a calculation, which mutual fund has the largest probability of a return
greater than 50%? Explain in words how you know.

d. The return on the “Domestic Small Caps” fund falls below what level with a probability of
95% ?