Measuring the Benefits of Option Strategies

For Portfolio Management

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1. Introduction

Derivative securities feature two important characteristics: financial leverage and asymmetric payoffs. Their leverage can be exploited to generate a cost-effective means of hedging risk, or of placing large bets on the movements of a wide range of variables including stock indexes, exchange rates, and interest rates. Several derivatives-related fiascoes of the 1990’s, including Orange County, Baring’s Bank, and more recently Long Term Capital Management, illustrate the fallout of leveraged bets gone bad. These events have imparted on derivatives a certain degree of notoriety and perhaps have diverted attention from the benefits of derivatives, especially those related to their unique payoff structure.

One application of derivatives in particular warrants discussion: equity options can provide mutual fund managers with efficient tools to alter a portfolio’s return distribution, and in this manner better meet the needs of investors. Options have asymmetric payoffs. Call options generate income when the price of the underlying stock rises above the option’s exercise price, whereas put options generate income when the price of the underlying stock falls below the option’s exercise price. The asymmetric payoff structure of options allows fund managers to change the shape of portfolio return distributions in optimal ways. In the parlance of risk management, fund managers can reduce downside risk while preserving or enhancing upside potential.

This report discusses the potential benefits of option strategies for portfolio management and investigates appropriate methods of quantifying them. First, we explain precisely how positions in options can add value to an equity portfolio. We argue that
investors naturally prefer portfolios with asymmetric return distributions. In particular, investors desire positive skewness in returns, meaning that the optimal distribution has a very low probability of extremely bad outcomes. Then we demonstrate that equity options are an effective tool for altering a portfolio’s return distribution so that it features the desired shape. Second, we examine existing mutual fund performance appraisal techniques, and identify appropriate methods for measuring the benefits of using options. We show that the Value at Risk paradigm at the core of corporate risk analysis is useful in a portfolio management context as well. Further, we propose a reward to risk ratio that distinguishes between upside and downside variability in a return distribution.

The rest of this report is organized as follows. Section 2 draws from utility theory to explain why investors prefer return distributions that exhibit positive skewness. Section 3 reviews the literature on mutual fund performance appraisal, and proposes a methodology that is appropriate for evaluating the benefits of using options. Section 4 describes simulation-based methods for measuring the risk and return of portfolios with positions in options. Section 5 concludes.

2. Theoretical Motivation for Managing the Distribution of Returns

In order to demonstrate the benefits of option strategies for portfolio management, we must establish what type of portfolio investors prefer. Utility theory provides some insight. Two well-accepted features of investor attitudes towards wealth are (1) investors prefer more to less, and (2) investors dislike uncertainty. From a portfolio management perspective, this result implies that investors accept higher levels of variance only when
rewarded by higher levels of expected return. Additional assumptions about investors' utility functions provide more detailed guidance regarding their preferences for mutual fund return distributions.

Arditti (1967) presents an argument that rational investors with reasonable utility functions should prefer positive skewness in the distribution of investment returns. A preference for positive skewness simply means that investors favor variation in the upside but not in the downside. A symmetric distribution with no skewness is compared to a skewed distribution in figure 1. Note that the positively skewed distribution has more weight in the positive tail. This implies that extremely positive outcomes are more likely than extremely negative outcomes, e.g. a portfolio's rate of return is more likely to be extremely high than extremely low.

To see mathematically why investors prefer positive skewness, suppose that an investor’s utility function $U$ is dependent only on her wealth. As pointed out by Kraus and Litzenberger (1976), Arrow (1971) suggests that a rational investor’s utility function for wealth likely exhibits three characteristics, (1) positive marginal utility, $U'' > 0$, (2) decreasing marginal utility, $U'' < 0$, and (3) non-increasing absolute risk aversion, $[-U''/U']' \leq 0$. As argued by Arditti (1967), these three characteristics are sufficient to establish that $U'''' > 0$, e.g. a preference for positive skewness in asset returns. This follows since $[-U''/U']' = [-U''U'''' + (U'')^2]/(U'')^2 \leq 0$ implies that $U'''' \geq (U'')^2/U'' > 0$.

Investor preference for skewness motivated a number of studies in the early 1970’s that incorporated skewness and higher order moments of securities returns. Since then, however, relatively few tests of asset pricing models have incorporated skewness. Notable exceptions exist. Kraus and Litzenberger (1976) present empirical evidence that
supports a market equilibrium model that includes skewness. Friend and Westerfield (1980) find less evidence in support of the three-moment model, but Lim (1989) verifies the Kraus and Litzenberger (1976) result. More recently, Harvey and Siddique (1998) find that conditional skewness is priced in the market.

From a corporate risk management perspective, preference for skewness in the distribution of cash flows has a natural motivation: the avoidance of financial distress. Suppose for example that a petroleum company with no inventory offers long term contracts to supply heating oil at a fixed price. If the future spot price is lower than the fixed price, the company can purchase heating oil at the relatively low market price and sell at the agreed upon fixed price. In that case, the supply project will generate large positive cash flows. On the other hand, if the future spot price of heating oil is higher than the fixed price, the supply project will generate large negative cash flows, which could drive the company to bankruptcy. Suppose that the company can purchase call options on heating oil. Now when the future price of heating oil is higher than the fixed price, the company can use the call options to purchase the heating oil at the options' exercise price. In this manner the cash flow distribution of the project can be changed from symmetric to positively skewed. In other words, the lower tail of the distribution can be eliminated, thereby reducing the risk of bankruptcy.

Portfolio managers can employ a variety of option strategies to alter the distribution of portfolio returns in a manner analogous to the corporate use of derivatives described above. Suppose for example that a mutual fund has reached a target level of return by November of some year, perhaps triggering a bonus for the fund manager. The manager has the incentive to reduce or eliminate the probability of negative returns for
the remainder of the year in order to preserve the current year gains. This can be accomplished via index put options, which generate income only when the stocks in the portfolio fall in value. A more detailed analysis of this example is presented in section 4.

In summary, reasonable assumptions about investor attitudes towards wealth lead to a preference for positive skewness in either wealth levels or portfolio returns. Furthermore, as suggested above, options provide fund managers with an effective means of generating positive skewness. Options will only be used, however, if their benefits are captured or quantified in performance measures. As shown in the next section, standard approaches to measuring mutual fund performance focus on the mean and variance of returns and ignore skewness. A performance measure that fails to reward positive skewness will not fully capture the benefits of option strategies, since one attractive feature of options is their ability to add positive skewness to a portfolio's return distribution. Appropriate measures of the benefits of using options must therefore incorporate changes in skewness attributable to them.

3. Measuring the Performance of Mutual Funds that Use Options

This section reviews existing methods for measuring the performance of mutual funds and identifies methods that can capture the benefits of using options in portfolio management. Most existing performance measures are based on a mean-variance framework that underlies popular factor models of asset returns. As argued in the prior section, however, investors will likely have preferences for distributional characteristics beyond mean and variance. A useful way of categorizing existing performance measures,
therefore, is to determine whether or not they reward positive skewness. Only those measures that look beyond the mean and variance of returns will be able to demonstrate the full impact of option strategies on mutual fund performance.

3.1. Performance Measures Based on Mean and Variance

A. Sharpe Ratio

One popular measure of performance relies only on the historical distribution of returns generated by an investment. Sharpe (1966) simply measures the reward-to-risk ratio as

\[
SR = \frac{\mu - R_f}{\sigma},
\]

where the numerator is the expected return of the investment less the riskless rate of interest and the denominator is the standard deviation of the return. The mean and standard deviation are typically estimated from sampling historical returns. A fund manager who either purchases undervalued stocks or sells overvalued stocks will increase the return of a portfolio without affecting risk, hence the Sharpe ratio captures stock selection ability. In addition, since the denominator measures total variation in returns, the Sharpe ratio captures the benefits of diversification. The advantage of the Sharpe ratio is that it is not subject to model misspecification. The risk measure in equation (1) captures total risk - including risk due to sensitivity to unspecified factors as well as investment-specific risk that is not priced in asset pricing models. The disadvantage to the Sharpe ratio is that it is an accurate measure of the risk-reward tradeoff only for those
investors who care about the first two moments of returns exclusively. In other words, the Sharpe ratio does not reward positive skewness.

**B. Jensen's Alpha**

Jensen’s (1969) test is based on the unconditional CAPM, which implies that all assets available to fund managers must satisfy the following:

\[ r_{i,t} = \beta_i r_{m,t} + u_{i,t}, \]
\[ E(u_{i,t}) = 0, \]
\[ E(u_{i,t} r_{m,t}) = 0, \] (2)

where \( r_{i,t} \) is the return of asset \( i \) between times \( t-1 \) and \( t \), \( r_{f,t} \) is the return of a riskless asset between times \( t-1 \) and \( t \), \( r_{i,t} = R_{i,t} - R_{f,t} \) is the excess return of asset \( i \), and \( r_{m,t} \) is the excess return of the market. A fund manager’s portfolio is a linear combination of assets satisfying equation (2), hence portfolios also satisfy equation (2) according to the CAPM.

Tests of stock-selection ability are usually conducted via least squares regression:

\[ r_{p,t} = \alpha_p + \beta_p r_{m,t} + \varepsilon_{p,t}, \] (3)

where the intercept, or “alpha,” indicates abnormal returns.

Jensen's alpha rewards stock selection ability by identifying the average unexpected return of the portfolio. Unexpected returns are generated by firm-specific risks that are not reflected in CAPM expected returns. If a manager has the ability to select stocks with unexpected positive returns, the manager will generate a positive alpha. The disadvantages of Jensen's alpha are (1) it does not reward a portfolio for being well diversified, and (2) it does not reward positive skewness.
3.2. Performance Measures that Incorporate Skewness

The Sharpe ratio and Jensen's alpha are appropriate only when investors restrict attention to mean and variance of returns. Alternative performance measures reward positive skewness. Rewarding skewness is particularly relevant for performance measurement when options are used, since one of the main benefits of options is their ability to change the shape of a portfolio's return distribution.ii

A. Market Timing

Market timing refers to the allocation of capital among broad classes of investments, usually restricted to equities and government bills or bonds. The successful market timer increases the portfolio weight on equities prior to a general rise in the market index, and shifts capital to government securities prior to a general fall in the market index. As described by Treynor and Mazuy (1966), market timing induces a non-linear portfolio return distribution. In particular, successful market timers will generate a positively skewed distribution whereas unsuccessful market timers will generate a negatively skewed distribution.

Treynor and Mazuy (1966) use the following regression to test for market timing:

\[ r_{p,t} = \alpha_p + \beta_{1p} r_{m,t} + \beta_{2p} r_{m,t}^2 + \epsilon_{p,t}, \]  

where \( \beta_{2p} \) measures timing ability. Note that this model is identical to the CAPM regression listed in equation (3) with the addition of a squared excess market return. The intuition is that successful market timing generates a portfolio return that is a nonlinear function of the market’s return, and the nonlinearity is captured by the squared excess return.
market return in equation (4). The resulting return distribution has positive skewness, since it includes outcomes of extremely high returns relative to an untimed portfolio and excludes outcomes of extremely low returns. Thus if the parameters of the regression in equation (4) are estimated using the historical returns of a portfolio, a positive estimate of \( \beta_{2p} \) indicates positive skewness.

Merton and Henriksson (1981) develop a different test of market timing. In their model, the mutual fund manager forecasts the future market return, as before, except now the manager decides between a small number (usually two) of targets for fund exposure. The model with two target betas can be tested via the following regression:

\[
    r_{p,t} = \alpha_p + \beta_{1p} r_{m,t} + \beta_{2p} \max(0, r_{m,t}) + \epsilon_{p,t}. \tag{5}
\]

The magnitude of \( \beta_{2p} \) measures the difference between the target betas, and is nonzero for a manager that times the market.

Market timing models capture asymmetries in a portfolio's return distribution via the convex explanatory variables included in the RHS of equations (4) and (5). As noted by Jagannathan and Korajczyk (1986), investing in options can also generate a skewed distribution that will be detected by market timing regressions. Thus a regression similar to the two described above could be used to measure the impact of option strategies on the return distribution of a managed portfolio. There are two disadvantages to this approach, however. First, as noted by Jagannathan and Korajczyk (1986), it is impossible to distinguish between the effect of market timing and the effect of investments in options. Second, regression models are estimated in a backward-looking fashion using historical data. The one sample path that occurred largely determines the performance
measurement. A more meaningful measure of the impact of options would be forward-looking and would incorporate the full spectrum of possible outcomes.

**B. Performance Analysis of Options**

Two studies by Evnine and Rudd (1984) and Galai and Geske (1984) present asset pricing models that could be used to measure performance when investments have option-like features. In both cases the general idea is to include the sensitivity of the option's return to the stock's return as an explanatory variable. The disadvantage of this approach is that the focus is on rewarding managers for having option selection ability, i.e. for buying undervalued options and selling overvalued options. As argued in Section 2, mutual fund managers should also be rewarded for investing in correctly priced options if those options are used to change the shape of the portfolio return distribution in an optimal way.

**C. Semi-variance Models**

Another way to capture preference for positive skewness is to separate downside variation from upside variation when quantifying risk. Fishburn (1977) works with the following general function to compute asymmetric variation:

\[
F_\alpha(t) = \int_{-\infty}^{t} (t-x)^\alpha dF(x)
\]

where \(F_\alpha(t)\) is the measure of asymmetric variation, \(F\) is the distribution of returns, \(t\) is a fixed "target" level of return, and \(\alpha\) is a positive constant. Fishburn calls this the \(\alpha\)-\(t\) model. When \(\alpha = 2\), the integral in equation (6) represents a weighted probability of the
outcome $x$ falling below the target, where the weights are the squared difference from the target. For this reason, the special case of $\alpha = 2$ is called the "semi-variance" of a distribution. Fishburn notes that Markowitz (1959) suggested this measure, and that prior studies have shown that semi-variance corresponds to criteria of choice reported by investment managers.

Semi-variance is potentially useful as a measure of the benefits of options for portfolio management because it captures the size of the lower tail of a return distribution. A well diversified portfolio that includes a position in index put options, for example, will have a substantially lower semi-variance than a corresponding portfolio without options since the puts generate income and reduce portfolio losses when the market return is low.

Bawa and Lindenberg (1977) develop an equilibrium asset-pricing model using Fishburn's $\alpha-t$ model, which they call a mean-lower partial moment model. They show that under appropriate assumptions about investors' utility functions, asset prices are determined in a fashion quite similar to the traditional Capital Asset Pricing Model. In particular, two-fund separation still holds, wherein investors optimally allocate all capital between the market portfolio and the riskless asset, and asset expected returns are linear in market expected returns. The difference is that the appropriate measure of risk is not covariance with the market portfolio, but the colower partial moment between the asset in question and the market. Harlow and Rao (1989) test the mean-lower partial moment asset pricing model and fail to reject it empirically, whereas the traditional capital asset pricing model is rejected.
Sortino and van der Meer (1991) label the semivariance measure "downside risk" and motivate it as variation below a "minimum acceptable return." Hagigi and Kluger (1987) develop their "Safety-First" performance measure in a similar spirit. The Safety-First model specifies an investor-defined "disaster" with some acceptable probability of occurrence. The performance of a portfolio or some other investment is measured by first determining whether or not the probability of the disaster occurring is greater or less than the acceptable level. For those that meet the criterion, rankings are based solely on expected return. For those that fail to meet the criterion, rankings are based on the probability of disaster, which takes into account both expected return and variance. The Hagigi and Kluger (1987) approach is quite similar to contemporary Value at Risk methods discussed below. Again, these methods are potentially useful to measuring the benefits of options since they explicitly incorporate the set of outcomes that are altered by option strategies.

Given the theoretical results of Bawa and Lindenberg (1977), we can construct a performance measure that extends the Sharpe ratio. The intuition is to reward portfolios with high expected returns and to penalize them not for total risk but for downside risk only. Let $DR$ denote the downside risk ratio:

$$DR = \frac{\mu - r_f}{\sqrt{F_2(t)}}.$$  \hspace{1cm} (7)

In effect, the $DR$ rewards positive skewness by reducing the denominator of the performance ratio for those funds that exhibit positive skewness. Consider, for example, two portfolios with identical average excess returns and identical standard deviation. One portfolio has a symmetric distribution, whereas the other has positive skewness. The two
will have identical Sharpe ratios, but the skewed portfolio will have a higher downside risk ratio than the symmetric portfolio.

The only unspecified parameter in $DR$ is the target return. One sensible choice is the riskless rate of interest. Another is the market index return, since that represents a benchmark for mutual fund returns.

D. Value at Risk (VaR)

Contemporary risk management addresses skewness preference by focusing on the probability of various levels of downside outcomes. This approach is widely known as Value at Risk or VaR. As described by Hull and White (1998), VaR calculations are used to make statements of the form "We are $\lambda\%$ certain that we will not lose more than $V$ dollars in the next $N$ days." $N$ represents the time horizon and $\lambda$ represents the confidence with which the statement is made. The value of $V$ is determined by the distribution of underlying variables and the associated payoff function. Duffie and Pan (1997) describe techniques for making VaR calculations.

Value at Risk is a useful corporate risk management tool because it assigns a probability to particularly important "lower-tail" outcomes, such as the level of cash imbalance that triggers financial distress. VaR calculations also are useful in testing the performance of alternative risk management strategies, such as hedging with derivatives. It stands to reason that VaR would be a useful measure of the benefits of derivatives-usage for fund managers. Suppose, for example, that a portfolio manager wishes to lock in year-to-date portfolio returns. He considers two courses of action: do nothing or invest in an appropriate option strategy to hedge future portfolio values. Using VaR, the
manager could compute the probability of negative returns for the remainder of the year both with and without an appropriate option strategy in place.

E. Summary

This section reviews existing measures of performance with the goal of identifying those that are appropriate for capturing the benefits of options in the context of portfolio management. The Value at Risk paradigm appears to be particularly useful in this regard because it can be used to answer specific questions about how options can affect the return distribution of portfolios. More specifically, VaR analysis can be used to assess how options alter the level of skewness or semi-variance in a portfolio's return distribution. As argued previously, these are questions with which investors are concerned. In order to apply VaR in a portfolio management context, we require a method of assessing the changes in the distribution of portfolio returns attributable to some candidate options-based strategy. We turn next to a discussion of this topic.

4. Simulation-based Performance Measurement

In order to measure the benefits of using options to manage risk in a portfolio, we must be able to meaningfully compare the performance of a portfolio with no options to the performance of another portfolio that incorporates option strategies. This section describes two approaches to conducting this type of comparison. The first uses historical fund returns that are then altered by the hypothetical inclusion of positions in options. This approach answers the question "What would this portfolio's return have been had it
included options?" The second approach constructs the entire distribution of future returns that a portfolio might generate with and without options. This approach answers the question "What is the likelihood that this portfolio's return will fall between \( x \) and \( y \) over the following year?" The distinction is important. The results of the first approach depend on the particular set of outcomes that occurred in the historical sample, hence it is backward-looking. More important to investors and fund managers is the set of outcomes that might occur in the future, which is precisely what the second approach describes.

4.1. Historical Simulation

Merton, Scholes, and Gladstein (1978, 1982) conduct two studies to measure the profitability of various options-based trading strategies. In the first, they compare holding period returns of investments in 136 stocks to alternative investments involving call options on the stocks. The strategies are compared using summary statistics on the positions, such as average return, standard deviation, and the maximum and minimum returns. Strategies involving options are shown to change the cross-sectional distribution of returns generated by the stock-only positions. In the second study, the authors perform similar experiments involving put options on stocks. Together, the papers serve to illustrate how several basic trading strategies involving options can aid investors in altering the risk-return profile of equity investments to better suit their needs.

Gastineau and Madansky (1979) criticize the type of analysis reported by Merton, Scholes, and Gladstein for not properly accounting for risk. Suppose, for example, that asset prices were determined by the Capital Asset Pricing Model. A fund manager
maintains a portfolio beta of 1 and the fund generates a return equal to the market return, as expected. Now if the fund manager were considering investing in options, perhaps index call options to enhance return, it would be an unfair to compare the fund's historical return to what the return would have been had a position in call options been included in the portfolio. The reason for this is clear - the fund manager would have altered the riskiness of the portfolio with the inclusion of options, hence one would have expected a difference in fund return. A meaningful comparison must take into account changes in risk.

Gastineau and Madansky (1979) suggest an alternative way to measure the profitability of a hypothetical options position. They propose to compare the implied volatility of the option at the time of the initial purchase or sale to an ex-post estimate of the volatility of the underlying asset over the option's life. Option price is proportional to volatility. If realized volatility were higher than the initial implied volatility, then the option was initially undervalued, and vice versa.

A fair comparison between alternative investment strategies certainly must account for the risk-return trade-off. However, the method proposed by Gastineau and Madansky (1979) shares another shortcoming with the analysis in Merton, Scholes, and Gladstein. Both techniques look backward in an effort to comment on strategies that investors or fund managers are considering today in anticipation of future payoffs. A more meaningful assessment of risk and return would comment on the entire range of future payoffs to particular strategies, and the relative probabilities of each payoff. In other words, effective risk management requires knowledge of the entire distribution of future payoffs.
Assessing the distribution of future payoffs is a central theme in contemporary corporate risk management. In particular, Value at Risk seeks to determine the probability that cash flows or investment values will fall within certain ranges over some time horizon. In order to compute such a probability, the distribution of future values must be estimated. A robust and flexible method for estimating distributions of payoffs or returns is Monte Carlo simulation.

4.2. Monte Carlo Simulation

The goal of Monte Carlo simulation is to represent all possible values that a variable can take over some period of time in the future, as well as the probability with which each outcome will occur. In other words, Monte Carlo simulation represents the distribution of the future values of a variable. There are two steps in conducting Monte Carlo simulation. First, one must establish an appropriate data generating process. A data generating process is appropriate if it is consistent with the true stochastic process governing changes in the variable of interest. For example, under the Black-Scholes (1973) assumption of geometric Brownian motion, continuously compounded returns are normally distributed. The decision regarding which data generating process to use is usually made with the guidance of statistical tests and historical data.

The second step in Monte Carlo simulation is to use the chosen data generating process to simulate a large number of future values of the variable of interest. To simulate stock prices consistent with the Black-Scholes assumption, for example, random numbers (or deviates) are drawn from a normal distribution. These deviates are then used to
compute future values. In order to reflect accurately all possible future paths, many simulations are required. The advantage of simulation methods is that they can simulate data consistent with any underlying stochastic process. In addition, the impact of options with payoffs equal to an arbitrarily complex function of the underlying variable can be easily included in the simulations.\textsuperscript{iii}

To illustrate, suppose an equity fund manager's portfolio generates continuously compounded returns, under the assumption of full reinvestment of dividends, that are normally distributed with annualized mean of 15% and annualized volatility of 15%. The fund's equity holdings closely match an equity index. The current value of the fund is $1 billion. The fund has returned 30% year-to-date, and the manager wishes to eliminate downside risk over the next month by purchasing European index put options that expire in one month. Suppose the current index level is 1,000, the put options have an exercise price of 980, and that put contracts have a multiplier of 500. Since each contract allows the sale of $500,000 of stocks, the fund manager considers the purchase of 2,000 put contracts. Suppose the riskless rate of interest is 5% annually, and that an investment in the index pays a continuous annual dividend yield of 3%. According to the Black-Scholes formula, the price of each put option contract is $4,191.36, hence the total investment in options would be $8,382,727, or about .8% of assets.

The impact of the put protection on the portfolio values at the end of one month are assessed via Monte Carlo simulation. Ten thousand normal variates were generated to represent the distribution of returns generated by the portfolio over a one month horizon, hence the normal distribution has a mean of 1.25% and a standard deviation of 4.33%. Two sets of portfolio values were then computed. The first set models the future portfolio
values with no put protection. Let $r$ denote one of the randomly generated monthly returns. The corresponding portfolio value (in millions) is $1,000e^r$. The second set of values models the future portfolio values with put protection. These are computed as $991e^r + \text{Max}[980 - 1,000e^r, 0]$. Figure 2 shows the resulting histograms. The put protection has clearly changed the shape of the distribution of the future portfolio values by eliminating the "lower tail" outcomes. Table 1 lists some summary statistics for the distribution of future values of the portfolio. The position in puts lowers the maximum portfolio value and the average portfolio value, since the price paid for the puts factors in to every simulation. However, the puts dramatically increase the minimum portfolio value from $861$ to $972$ million. Without the puts, over 16% of the simulations result in a portfolio value of less than $970$, whereas none of the put protected portfolio values fall below this amount.

Table 2 lists corresponding statistics for the rate of return of the portfolio over the next month. Note that the Sharpe ratio of the unprotected portfolio is about 20% higher than the put-protected portfolio, reflecting the decrease in return generated by the expense of the put position. By this measure the value of the puts seems questionable. However, consider the downside risk ratios with a target of 0%. The put-protected portfolio exhibits a downside risk ratio almost two times the unprotected portfolio, reflecting the reduction in variability below the target return. Thus the $DR$ captures the skewness generated by the option position, whereas the $SR$ does not.
4.3. Other Issues

Monte Carlo simulation is a flexible technique for estimating the distribution of future portfolio payoffs, and, for this reason, is a useful method for illustrating the impact of option trading on mutual fund performance. Several issues related to the practical application of options-based strategies are relevant.

Perhaps the most important practical detail that should be included in a simulation analysis is the impact of taxes on portfolio payoffs. Fortunately, simulation techniques are flexible enough to incorporate taxation schemes of arbitrary complexity. The computer program written to generate random portfolio payoffs (consistent with a model of underlying stock returns) can also include an algorithm for determining the tax liability of a particular option position in each simulation. The result then would be an illustration and analysis of after-tax portfolio payoffs.

When index options are analyzed, as they were in the example of section 4.2, another practical issue is the correlation between the managed equity portfolio and the index portfolio underlying the option. The example made the simplifying assumption that the equity portfolio matched the equity index underlying the option. This way one variable was simulated - the future value of the index portfolio - and this was used to assess both the value of the equity portfolio and the payoff of the option. In practice, actively managed mutual funds will differ substantially from the index portfolio. To address this, the joint distribution of the index portfolio returns and the actively managed portfolio returns can be specified for the simulation. Then, each simulation corresponds to a joint outcome of the distribution describing the index portfolio return and the
distribution describing the actively managed portfolio return. The index portfolio return determines the index option payoff, which is then combined with the actively managed portfolio return to illustrate the distribution of the actively managed portfolio with options included. The key to this approach is correctly specifying the correlation structure between the index portfolio and the actively managed portfolio.

Another practical detail to consider is whether index options or individual equity options are used. In the example in section 4.2, an index option was used to provide portfolio insurance. When individual equity options are used in a trading strategy, a multivariate simulation can be used to model the impact of the option payoffs on the portfolio return distribution. Suppose for example that a fund manager wishes to protect a portfolio value from declines in one particular stock in which the fund is heavily invested. The proper simulation requires the specification of the joint data generating process governing the portfolio return and the individual equity return. This approach can be extended to the case when many individual equity options are used in a trading strategy.

Lastly, one limitation of Monte Carlo simulation is worth noting. By construction, simulation methods can model payoffs of European-style options but not American-style options. The reason is that option positions are assumed held to maturity, since is computationally burdensome to model the early exercise decision within a simulation. To the extent that the early exercise feature is important, ignoring it will tend to bias downward the return to an option strategy. However, when out-of-the-money options are used, the early exercise feature generally is not significant, so that the simplification should not substantially affect the results of a simulation.
5. Summary

The purpose of this report is to identify appropriate performance measures for portfolio management when options are used to manage risk. Existing methods have several shortcomings. The Sharpe ratio and Jensen's alpha, for example, do not reward the asymmetric payoffs that investors prefer. Other methods, such as the factor models of Evnine and Rudd (1984) and Galai and Geske (1984) focus on performance due to identification of mispriced options. Prior simulation experiments, as in Merton, Scholes, and Gladstein (1978, 1982) illustrate how option strategies could have affected on particular set of portfolio returns, but do not provide a full picture of the impact of options trading on the risk and reward profile of a portfolio.

An alternative approach based on Monte Carlo simulation estimates the entire distribution of future portfolio payoffs. This allows analysis similar to Value at Risk calculations used in corporate risk management. An example shows how Monte Carlo simulation can be used to provide a vivid illustration of the impact of options trading on portfolio returns. The distributions of portfolio returns under various assumptions about options usage can be compared. In addition, performance measures, such as the downside risk ratio developed in the paper, can be computed from the simulated portfolio values, further illustrating the benefits of options.
Figure 1. Symmetric and Skewed Distributions

Symmetric Distribution

Skewed Distribution
Figure 2. Put Protection

Depicted are the frequencies with which future values of two index portfolios, in millions, occur in a simulation experiment. One portfolio is protected by insurance provided by a position in put options.
Table 1. Summary Statistics of Portfolio Values

Listed are summary statistics of the distributions of future values of two index portfolios, in millions. One portfolio is protected by insurance provided by a position in put options.

<table>
<thead>
<tr>
<th></th>
<th>Unprotected</th>
<th>Put-protected</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max</td>
<td>$ 1,170.91</td>
<td>$ 1,161.09</td>
</tr>
<tr>
<td>Min</td>
<td>$ 861.12</td>
<td>$ 971.79</td>
</tr>
<tr>
<td>Average</td>
<td>$ 1,013.35</td>
<td>$ 1,010.33</td>
</tr>
<tr>
<td>Median</td>
<td>$ 1,012.52</td>
<td>$ 1,004.03</td>
</tr>
<tr>
<td>Std. Error</td>
<td>$ 43.87</td>
<td>$ 35.87</td>
</tr>
<tr>
<td>Pr(FV&lt;970)</td>
<td>16.34%</td>
<td>0.00%</td>
</tr>
</tbody>
</table>
Table 2. Summary Statistics of Returns

Listed are summary statistics of the distributions of returns of two index portfolios. One portfolio is protected by insurance provided by a position in put options.

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<tr>
<th></th>
<th>Unprotected</th>
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<tbody>
<tr>
<td>Max</td>
<td>15.78%</td>
<td>14.94%</td>
</tr>
<tr>
<td>Min</td>
<td>-14.95%</td>
<td>-2.86%</td>
</tr>
<tr>
<td>Average</td>
<td>1.23%</td>
<td>0.97%</td>
</tr>
<tr>
<td>Median</td>
<td>1.24%</td>
<td>0.40%</td>
</tr>
<tr>
<td>Std. Error</td>
<td>4.33%</td>
<td>3.50%</td>
</tr>
<tr>
<td>SR</td>
<td>0.19</td>
<td>0.16</td>
</tr>
<tr>
<td>DR</td>
<td>0.34</td>
<td>0.58</td>
</tr>
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References

Gastineau, G. and A. Madansky, 1979, Why simulations are an unreliable test of option strategies, *Financial Analysts Journal* September-October, 61-76
Harvey, C. and A. Siddique, 1998, Conditional skewness in asset pricing tests, working paper, Duke University
Hull, J. and A. White, 1998, Value at risk when daily changes in market variables are not normally distributed, *Journal of Portfolio Management* Spring, 9-19


Endnotes

i See Kraus and Litzenberger (1976) for references.

ii See Bookstaber and Clarke (1981) for a discussion of this topic.

iii The classic paper on the simulation approach to valuing options is Boyle (1977).