On the competitive effects of bidding syndicates*

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Abstract

Firms commonly form syndicates to bid jointly for financial assets. Recently, this practice has come under legal scrutiny motivated by models which suggest syndicates may be anticompetitive. These models do not account for two important features of financial markets: bidders’ value estimates are likely to be correlated with each other, and complicated mechanisms known to be optimal in such settings are eschewed in favor of simple auction formats. We show that these features make it possible for syndicate bidding to generate higher revenues for the auctioneer, even when syndicates lead to a highly concentrated market.

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1 Introduction

In 2006, the Department of Justice (DOJ) opened an investigation into the way private equity firms bid for takeover targets. At issue are “club deals” in which several private equity firms form syndicates to submit a joint bid. Concurrently, shareholders of takeover targets sought damages in private civil lawsuits. Naming major private equity firms as defendants, shareholders contended that they were “deprived of the full economic value of their holdings,” receiving artificially reduced prices as club deals dampened competitive pressures that would exist if firms bid separately.\(^1\) Initial public offering syndicates were sued on similar grounds.\(^2\) According to Securities and Exchange Commission (SEC) commissioner Paul Atkins, “This suit seeks to trump the securities laws with the antitrust laws” arguing that an unfavorable ruling “could devastate America’s process of capital formation, wreak unprecedented havoc” and jeopardize “the stability in our capital markets” (Atkins 2006).\(^3\)

The alarm on the part of the SEC reflects a significant difference in perspective between the SEC and the DOJ. Syndicate bidding is pervasive in financial markets, from angel investors (Sohl 1999, May 2002) and venture capital firms (Lerner 1994) to underwriting of primary equity (Corwin and Schultz 2005) and commercial lending (Dennis and Mullineaux 2000). In recent years, a majority of venture capital funding and nearly half of large private acquisitions were by syndicates rather than single firms (Lai 2005, Berman and Sender 2006). Conversely, “syndication” among bidders is often seen as a euphemistic expression for collusion by the DOJ, which pursues more criminal convictions for bid-rigging than for all other market conspiracies combined (Froeb and Shor 2005).

The portfolio diversification perspective suggests that syndicates provide capitalization and risk reduction (Wilson 1968, Chowdhry and Nanda 1996, Stuart and Sorensen 2001, Lockett and Wright 1999), often pointing to deals that could not have happened in the absence of joint bidding. Yet, the portfolio diversification justification for syndicates does not support their existence for small deals, and is becoming less persuasive in general as financial firms grow in size. For example, Chen and Ritter (2000) note the increasing size of modern financial firms, arguing “Today, there is little reason to form a syndicate to perform the traditional economic roles of risk sharing, distribution, and meeting capital requirements” (p. 1120). This raises the question of why syndicates persist in the current environment. Some have proposed competition-reducing motivations as one possible answer. A noted antitrust attorney warned “If a bidder or group was able to bid on its own—but to avoid competition joined with other

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\(^3\)Especially notable is that these suits were brought under the Sherman Act which, along with the DOJ’s investigation, represent antitrust’s foray into the traditional purview of the SEC. In a recent ruling (Credit Suisse v. Billing), the United States Supreme Court specifically sidestepped this issue (holding that the general argument was not “preserved” as it was not sufficiently argued in the lower courts), opting instead for a “fact specific” decision (Baker and Ostrau 2007). This leaves open the future reconsideration of whether the SEC’s regulatory framework is sufficient, effectively immunizing all financial syndicates from antitrust review, or, conversely, if financial institutions will be judged by the same standards as candy stores and steel mills.
bidders capable of bidding on their own—that could be viewed as unlawful.4

In this study, we investigate the competitive effects of syndicates, and determine the effect on resulting prices when bidders form syndicates rather than participating individually. To fix ideas, suppose that six private equity firms all show interest in acquiring a corporation. All six are sufficiently well capitalized and risk neutral, and have equivalent abilities to manage the corporation if they succeed. That is, each is capable of bidding for the firm on its own, and assume that the actual value of the corporation will be the same regardless of the identity of the acquiring firm. However, firms do not know this value with certainty. Place yourself in the position of the target corporation’s major shareholder. To maximize the price the acquiring firm pays for your shares, would you prefer each bid separately, or the firms organize themselves into two syndicates, with each syndicate submitting only one bid?5

Conventional models find that syndication leads to more competitive bids, as syndicates form better estimates of value by pooling members’ information, but reduces the resulting price as the loss of competition leads to fewer overall bids (e.g., Bikhchandani and Riley 1991, Bulow and Klemperer 2002, Mares and Shor 2008a). The reduction in the number of competitors seems to dominate information pooling even when the seller is given full strategic latitude to respond to syndicated bidding by optimally altering the auction mechanism (Mares and Shor 2008b). These models confirm a common regulatory assumption that syndication is generally anticompetitive.

The results of these models are predicated on two assumptions. First, it is assumed that each bidder’s estimates of value are uncorrelated with each other. Second, the seller uses an optimal, revenue-maximizing mechanism. Both assumptions are unrealistic descriptors of financial syndicates. While the first assumption that bidders’ signals—their private information—are independent is very common in the auction theory literature, financial analysts’ signals are usually correlated with the ex post realized value and with each other (Brown and Richardson 1987, Barron, Byard, Kile, and Riedl 2002). A series of papers have established that even arbitrarily small correlation among signals allows the auctioneer to design a mechanism that extracts full surplus (Crémer and McLean 1985, 1988, McAfee and Reny 1992).6 Under this optimal mechanism, bidding firms earn zero profits whether or not they form syndicates, making moot the question of competitive effects. Yet, these optimal

4Lauren Albert (partner at Axinn, Veltrop, & Harkrider), cited in Berman and Sender (2006). Another attorney attributed the aforementioned legal investigations to a theory “that the formation of consortiums dampens competition in auctions because sponsors who would otherwise be bidding against each other team up to jointly bid and drive down the sale prices” (Schwartzman 2006, p. 99). Beyond capitalization and risk-sharing, researchers have examined many other motivations for forming syndicates, including relationship networks (Hochberg, Ljungqvist, and Lu 2007), monitoring and moral hazard (Pichler and Wilhelm 2001), coordination costs (Wright and Robbie 1998), as well as window dressing, certification and marketing, and access to distribution networks.

5In another context, would a firm considering an equity offering realize lower or higher underwriting fees if it limited the power of competing investment banks to form underwriting syndicates?

6Full surplus extraction requires affiliation, a stronger concept than correlation, loosely defined as “local positive correlation everywhere” (Klemperer 2004, p. 48).
mechanisms are quite complex, exhibit significant implementation challenges, are very sensitive to small variations in the model, and have never been adopted in practice. Crémé and McLean (1988) note that these mechanisms place unrealistic requirements on the auctioneer and bidders, suggesting that “less profitable but vastly simpler auctions [are] used in practice” (p. 1254). The question of how syndicates impact typical auction mechanisms (such as ascending and sealed bid auctions) despite their suboptimality is the subject of this paper.

When values are correlated and the auctioneer uses a typical auction, we identify, along with information pooling and loss of competition, a third effect of syndication. Syndicates change the industry’s information profile—the distribution of available information among active bidders. Syndicates can also move the industry to a profile more suitable to the auction format in use. A typical auction’s revenue could be far from optimal in the absence of syndicates but the same auction format could perform substantially closer to the optimum when information is concentrated among fewer, but larger syndicates. We term this phenomenon the mechanism effect. We identify conditions under which the mechanism effect dominates the loss of competition, causing syndication to raise the auctioneer’s revenue. In these cases, the objectives of antitrust and securities regulation with respect to syndicated bidding are not brought into direct conflict. The effect of syndication depends on the selling format used (e.g., sealed bid or open auction) and the relationship between bidders’ information and the asset’s actual value. We show that syndication can be competitive only in sealed-bid auctions, and not, for example, in English auctions in which bidders update their beliefs about the asset’s value upon observing other bidders drop out.

Whether syndicates increase or reduce competition is not merely a regulatory question. For the owners of financial assets, firms with equity offerings, or potential takeover targets, allowing or prohibiting syndicate bidding is a strategic decision with serious consequences. It is not uncommon for firms to stipulate limitations on syndicates as part of their confidentiality agreements governing the disclosure of financial information. By precluding syndicates, these firms may be costing their shareholders potential gains.

In the next section, we introduce the model and then demonstrate that, in auctions for a single item, syndicates can raise or lower the resulting price depending on the auction format and the value function. A value function maps individual bidders’ estimates of the object’s value into a common estimate. Robustness of our result is examined in three dimensions.

7These mechanisms require risk-neutrality and unlimited budgets and liability, since these mechanisms may require firms to take risks with arbitrarily large amounts of capital. A theoretical critique is that a bidder’s signal about the asset’s value is also the source of his beliefs about his opponent’s information. Furthermore, bidders’ participation in full-surplus extracting mechanisms is in question whenever small information acquisition and bidding costs are present (Harstad 2005). Arguments on theoretical grounds argue that the full-surplus extraction result is not generic (Neeman 2004, Heifetz and Neeman 2006).

8Typical language to this effect in confidentiality agreements reads “For a period of one year, [the potential bidder] ... will not participate in or encourage the formation of any partnership, syndicate, or other group which seeks to acquire ownership of any Securities in or which seeks to affect control of [the target].” Both FindLaw and SEC Info online have multiple examples.

9For example, if one was privy to the estimates of all bidders, would one rely more on the minimum, the
First, we consider how variations in the value function influence the magnitude of the mechanism effect. We show that the mechanism effect is sufficiently robust, inducing pro-competitive effects of syndicates under a broad class of value functions. Second, we ask how the mechanism effect relates to the asymmetry of syndicates’ information profiles. Certainly, 500 major firms forming one syndicate against a lone small firm may differ from two syndicates of 250 firms each. Third, we show that our results carry over to uniform-price, multi-unit auctions. These models are loosely descriptive of bond auctions and of competitive underwriter selection for equity offerings (Parlour and Rajan 2005). We conclude with implications and a simple screen in the form of a thought experiment which portends the effects of syndication in specific environments.

2 Model

A total of $n$ firms participate in an auction for an asset with a common value $v$. Firm $i$ privately observes a signal $s_i$. Denote by $s$ the vector of signal realizations. The value of the asset is given by $v(w,s)$, which is a function of the signals and a value parameter, $w$. Prior beliefs about $w$ are uniform on $[\underline{w}, \overline{w}]$. This assumption is intended to limit priors to the least informative case. Private signals are conditionally independent and uniformly distributed on $[w-\theta, w+\theta]$. This conditional independent structure ensures that signals are affiliated with each other and with the underlying value. The spread parameter $\theta$ controls both the precision of individual signals as well as their correlation. The variance of individual signals is unaffected by the realization of $w$, leaving the precision of private information independent of the true value.

We compare equilibrium bids and revenues under two scenarios, one without syndicates, in which the $n$ firms each place a competitive bid observing only their own signals, and one with a highly-concentrated information profile consisting of two syndicates with $n$ signals between them. Each syndicate, privy to all of the signals of its members, places a single bid. We consider the polar case of firms consolidating into two syndicates for several reasons. First, it is the most concentrated information profile that still allows for competitive effects in an auction. A single bidder obviously makes an auction moot. Therefore, any pro-competitive effects from forming only two syndicates are especially stark. Second, analytical solutions are possible in these cases, allowing for direct comparisons. Lastly, in intermediate cases, with multiple bidders each with multiple signals, existence of equilibria is not guaranteed (Jackson 2005).

\[^{10}\]In the text, we consider only cases where signals are in $[\underline{w} + \theta, \overline{w} - \theta]$. Signals in this interior range are unbiased, $E[w|s_i] = s_i$. Effectively, we assume that the distribution of $w$ has a diffuse prior; for any signal a bidder receives, the bidder does not know if that signal is higher or lower than that of other bidders. This can be operationalized by assuming that $\underline{w} \to -\infty$ and $\overline{w} \to \infty$ or, as we do here, by assuming that signals are sufficiently in the interior.
We consider two formulations of the value function. First, we examine as a benchmark case perhaps the most prevalent model of a common value auction with affiliated signals:

\[ v(w, s) = w \]  

(1)

Nature draws some value from an uninformative prior and then provides each bidder with a conditionally independent signal centered on this value.\(^{11}\) Second, we turn to more general value functions to understand which formulations make syndicates pro-competitive.

We consider both a second-price sealed-bid auction and an open ascending (English) auction. In the second-price auction, the highest bidder wins the asset, and pays a price equal to the second-highest bid. In an English auction, prices continuously rise with bidders indicating their willingness to pay the current price or choosing to publicly and irrevocably drop out. A bidding strategy maps one’s signals and the history of bidders who have dropped out into a price at which the bidder would drop out. In our context, English auctions weakly revenue-dominate second-price auctions which, in turn, dominate first-price auctions (Milgrom and Weber 1982a).\(^{12}\) In private-value settings, where bidders have idiosyncratic but known values for the asset, the English and second-price auctions are strategically equivalent. With correlated signals, the English auction provides additional information to bidders as they may draw inferences from other bidders’ dropping out of the bidding. We derive the equilibria and revenues when firms do and when they do not form syndicates.

3 The Benchmark Model

We first consider the value function given in Equation (1), where \( v(w, s) = w \). Because the signal each firm receives is an unbiased estimate of the object’s value, the expected value of the object equals one’s signal, \( E[v|s_i] = s_i \). However, a firm cannot generally bid this amount in equilibrium, as it does not account for the adverse selection inherent in selecting the auction’s winner. The bidder who wins learns that his signal is the highest of all bidders, and thus should condition his bid on this fact, bidding lower than one’s signal.

**Claim 1.** In a second-price auction, the equilibrium bid in the benchmark model without syndicates is given by:

\[ b(s) = s - \left( \frac{n - 2}{n} \right) \theta \]  

(2)

\(^{11}\)For example, this model has been used to study endogenous bidder entry (Harstad 1990), the role of experience in overcoming the winner’s curse (Kagel and Richard 2001), and the rationing of oversubscribed IPOs (Parlour and Rajan 2005). Its equilibrium properties when there are no syndicates are well known (e.g., Klemperer 2004).

\(^{12}\)First price auctions are particularly difficult to analyze in our set-up with multi-dimensional signals, especially under asymmetry. Numerical computations in DeBrock and Smith (1983) show that they yield qualitatively similar results to those we find in the second price format.
Claim 2. In an English auction, the equilibrium drop-out point in the benchmark model without syndicates is given by:

\[ b(s) = \begin{cases} 
  s & \text{if no one has yet dropped out} \\
  \frac{1}{2}s + \frac{1}{2}s_{\text{min}} & \text{otherwise}
\end{cases} \]  

where \( s_{\text{min}} \) is the price at which the first bidder dropped out.

The expected value of the asset conditional on all signals (if a bidder was made aware of them) is \( E[v|s] = \frac{1}{2} \min \{s_1, \ldots, s_n\} + \frac{1}{2} \max \{s_1, \ldots, s_n\} \). In a second-price auction, no winner’s curse correction is required when there are precisely two bidders (Milgrom and Weber 1982a), so one simply bids the expected value of \( v \) given one’s signal. For two syndicates, a similar result applies, in that each syndicate bids the expectation of \( v \) given all signals of its member firms. The English auction with two syndicates ends as soon as the first bidder drops out. Since a syndicate cannot glean any insight from the drop-out behavior of its only rival, bids in the English and second-price auctions are identical.

Claim 3. In second-price and English auctions with two syndicates, the equilibrium bid of a syndicate with \( m \) member firms and signals \( s_1, \ldots, s_m \) in the benchmark model is given by:

\[ b(s_1, \ldots, s_m) = \frac{1}{2} \min \{s_1, \ldots, s_m\} + \frac{1}{2} \max \{s_1, \ldots, s_m\} \]

Whether or not syndicates hurt competition can be decided by asking which information profile leads to higher expected revenues. We first consider competition among \( n \) firms compared to competition among two symmetric syndicates, each with \( n/2 \) members.

Proposition 1.

(i) In an English auction, an auction with two symmetric syndicates leads to lower expected revenue than an auction without syndicates.

(ii) In a second-price auction, an auction with two symmetric syndicates leads to higher expected revenue than an auction without syndicates.

Syndicates can have pro-competitive effects in the sealed bid auction but not in the open auction. These auction formats differ inherently in their ability to inform bidders about the asset’s value. An open auction reduces bidder uncertainty as bidders update their estimates each time a lower bidder drops out of the auction. The final price reflects not only the private information of the price-setting bidder but the inferred information of all bidders below him. Allowing for syndicates in this context does not improve on the information available to the price-setting bidder, but does reduce competition, leading to lower prices. Sealed bid auctions, in which bids are submitted independently and without knowledge of others’ bids, provide much less information about the object’s value. In these cases, syndicates substitute for the
information role of an open auction. By combining the signals of its constituent members, the pooling of information within syndicates substitutes for the lack of competition among syndicates.

In light of recent results on independent signals, which suggest that syndicates always reduce revenue (e.g., Waehrer and Perry 2003, Mares and Shor 2008b), our result may seem somewhat counter-intuitive. Past results show that under optimal mechanisms, the competition effect (which reduces the number of bidders) dominates the information pooling effect (which provides each syndicate with better information than individual bidders have). To understand our results, we consider these effects in turn.

Without syndicates, the equilibrium winner’s curse adjustment for a sealed-bid auction in Eq. (2) guarantees that the bid is significantly below one’s signal. As we increase competition, in large auctions, the winner’s curse correction will drive the bid down towards \( s - \theta \), while the increased competition implies more “draws” of signals from the distribution, and extracts surplus at the rate at which the second highest bid approaches the upper bound of the distribution, \( v + \theta \). These forces demonstrate the competition effect as the price converges to \( v \) asymptotically, effectively aggregating the disparate information held by bidders.

With only two syndicates, an increase in the number of original bidders does not intensify competition, but does improve the information available to each syndicate. The second price auction allows a syndicated bidder to bid, in Eq. (4), as if no winner’s curse problem exists. An increase in \( m \) leads very quickly to extremely tight estimates of the true value of the asset. Therefore, informational rents of the syndicates disappear rapidly since equally well-informed bidders are likely to place bids on a common-value asset that are very close to each other.

Another effect of bidding in syndicates comes from the equilibrium bids drawing information from both the highest and the lowest signals. Tying price to multiple sources of information generally improves auction revenues (Parlour and Rajan 2005, Mezzetti and Tsetlin 2006). The lowest signals contain as much information about the true value of the asset as do the highest signals, yet in the absence of syndicates, only the two highest signals (possessed by the winning and price-setting bidders) are incorporated into the price. In sealed bid auctions, these effects tip the balance in favor of syndicated bidding despite a loss of competition.

The revenue-improving role of syndicates in sealed bid auctions does not require symmetry, as the following proposition demonstrates.

**Proposition 2.** Consider \( n \geq 3 \). A second-price auction with two asymmetric syndicates (of sizes \( m \) and \( n - m \)) leads to higher expected revenue than a second-price auction without syndicates as long as the asymmetry is not too severe. That is, as long as

\[
0.276 \approx \frac{1}{2} \left( 1 - \frac{1}{\sqrt{5}} \right) < \frac{m}{n} < \frac{1}{2} \left( 1 + \frac{1}{\sqrt{5}} \right) \approx 0.724
\]  

The proposition indicates that even asymmetric arrangements of firms into syndicates can
be pro-competitive, as long as neither syndicate contains more than about 3/4 of the firms. The above bounds are not tight unless \( n \) is very large. The fewer the number of firms, the greater the level of asymmetry that still offers competitive improvements. With five or fewer firms, facing any two syndicates is more profitable for the auctioneer than independent bidders. In the appendix, we derive the exact bounds for any \( n \).

Asymmetric syndicates show the limits of the mechanism effect. While it is still true that information pooling within a syndicate substitutes for the lack of competition between syndicated bidders, the precision of a syndicate’s private information determines the revenue rankings. Imagine an extremely asymmetric situation in which a larger, almost precisely informed syndicate \((n - m \to \infty)\) bids against a small group \((m \text{ finite})\). Since the larger syndicate is perfectly informed about the object’s value (having an arbitrarily large amount of signals), the smaller syndicate will win when it overestimates the true value, and will lose (but set the price) when it underestimates the value. Since the prior distribution over signals is uniform and therefore symmetric, it is easy to see that each situation is equiprobable. This means that half the time the price will be determined by the less informed bidder. Thus, the price will be less informative than in the auction without syndicates. Our results point to this trade-off in terms of bounds on the level of asymmetry.

The benchmark model indicates that even if syndication reduces competition to two active bidders, we are still very likely to have positive, pro-competitive effects in environments in which the auction mechanism reveals only a single bidder’s private information. This result requires none of the traditional portfolio-theory justifications for syndicates; even if all firms have sufficient capital and risk tolerance to bid on their own, syndicates can still be pro-competitive. Of course, if syndicates introduce further efficiencies, this would only strengthen their desirability.

4 General Model

In our benchmark model, the maximum and minimum signals are sufficient statistics for the underlying value. In particular, no matter how many signals a syndicate possesses, its best guess about the value of the item is the average of its maximum and minimum signal. This property is not unique to this model as many non-uniform distributions of signals will yield identical equilibrium bid functions.

Define \( s_{(j)} \) as the \( j \)-th highest order statistic of the vector of private signals, \( s \). In this section, we follow Milgrom and Weber (1982a), allowing for a broader class of value functions

\[ h(x) = \frac{1}{2} \left( \frac{x}{w + \theta} \right)^{\frac{1}{2}} \] where \( h() \) is a power function.
which depend both on nature’s draw and on the realizations of private information. Consider value functions of the form
\[ v(w, s) = \gamma_0 w + \sum_{i=1}^{n} \gamma_i s_i \]
where \( \gamma_j \) are positive constants. Our benchmark model can be obtained by setting \( \gamma_0 = 1 \) and \( \gamma_i = 0, i \geq 1 \). As in our benchmark model, an English auction outperforms a second-price auction in this context without syndication. Since the two bidders with the highest signals would observe the drop-out points of bidders with lower signals, much of the uncertainty in the value function would be resolved and competed away between them. In this section, we concentrate on the sealed-bid second-price auction.

Not all parameterizations of this model are analytically tractable in the syndicated case, with both existence and uniqueness of equilibria in question (Mares and Harstad 2007). It still lends considerable insight into the generality of our results for the benchmark model. For the case without syndicates, we demonstrate in the next claim that the general value functions are bid and revenue equivalent to a simple model where
\[ v(w, s) = \alpha s_{(n)} + (1 - \alpha) s_{(1)} \quad (6) \]
for an appropriately chosen \( \alpha \). When \( \alpha = 1/2 \), this formulation is equivalent in bids and revenues to our benchmark model. Thus, even though the value function is different from our benchmark model (no \( w \) appears on the right hand side of Eq. 6), we obtain identical equilibria.

**Claim 4.** In an auction without syndicates, any linear (convex) combination of \( w \) and the order statistics of bidders’ signals is equivalent, in second-price equilibrium bids and revenue, to some linear (convex) function of only \( s_{(1)} \) and \( s_{(n)} \).

Working with the simplified form of the model allows for closed form solutions in the presence of syndicated bidders. We exploit this property to characterize the revenue effects of syndicated bidding for different values of \( \alpha \).

**Proposition 3.** Consider the value function in (6) and \( n \geq 4 \). There exists an \( \alpha^* < 1 \) such that two symmetric syndicates result in higher revenues than individual bidders in a second-price auction if and only if \( \alpha > \alpha^* \). Furthermore,

- \( \alpha^* \) is increasing in \( n \),
- \( 0.314 \approx \frac{1}{4} (7 - \sqrt{33}) \leq \alpha^* \leq \frac{1}{2} (3 - \sqrt{5}) \approx 0.382 \), and
- When \( \frac{1}{4} (7 - \sqrt{33}) < \alpha < \frac{1}{2} (3 - \sqrt{5}) \), two symmetric syndicates lead to higher revenues than independent bidders if and only if \( n < \frac{2\alpha}{\alpha^2 - 3\alpha + 1} \).
This result is consistent with our previous propositions, but allows us to explore the trade-offs between the information pooling, competition, and mechanism effects more directly. Consider the extreme case where \( \alpha = 0 \) so the value is equal to the highest signal received by any bidder. The equilibrium bid of a syndicate when only two syndicates exist is simply the maximum signal held by that syndicate. The competition effect is clearly the dominant force in this example since any concentration of information carries with it the chance that the two highest signals are in possession of the same syndicate. This means that, in the absence of syndicates, the second highest signal is the price, while with syndicates there is some probability that even lower order statistics will form the price. Syndicated bids will therefore lead to lower prices.

A different revenue dynamic is at work when \( \alpha = 1 \), so the value is equal to the minimum of the signals. The optimal mechanism in this case is actually relatively simple, involving a sealed bid auction where the highest bidder wins the asset and pays the lowest submitted bid (Mares and Harstad 2007). This mechanism is ex-post incentive compatible and extracts full surplus. The second price auction generally performs poorly in these circumstances (Mezzetti and Tsetlin 2006), but when exactly two syndicates compete, the second price auction becomes equivalent to the lowest-bid auction. The winner of the auction with two syndicates pays the minimum signal of the other syndicate, which is precisely the object’s value. The second-price auction extracts full surplus. Thus, a second price auction becomes increasingly closer to the optimum with fewer bidders, and extracts full surplus when only two syndicates exist. Facing a highly concentrated information profile with precisely two syndicates, the auctioneer captures greater revenue than if any number of bidders bid independently.

While the maximum-value auction (\( \alpha = 0 \)) displays the negative force of the competition effect, the minimum value auction (\( \alpha = 1 \)) highlights the positive, pro-syndicate aspects of the mechanism effect. Our result suggests that, for a majority of the parameter space, the mechanism effect dominates.

5 Multi-unit auctions

So far, we have assumed that there is one item for auction. In many settings, including underwriting and treasury auctions, bidders request both a price and a quantity. In the context of IPO book-building, Parlour and Rajan (2005) examine when a monopolist seller would introduce rationing, and ask if selling only a fraction of supply can increase total revenues. The answer is a qualified yes, providing some explanation for underwriters rationing in oversubscribed IPO markets. The model they employ is equivalent to our benchmark model, where \( v(w, s_1, \ldots, s_n) = w \), and \( n \) bidders with unit-demands compete in uniform price auctions for \( k \) units of identical assets.

Multi-unit auctions present additional analytical challenges in mechanism design. First, while we maintain the analytically tractable assumption that each firm initially has unit de-
mands (Bikhchandani and Fu Huang 1989, Parlour and Rajan 2005), syndicates will demand multiple units as they represent several firms. Second, one needs to resolve how an English and a second-price auction generalize to multiple units. For an English auction, the natural extension is an ascending auction which ends when the number of remaining active bidders is equal to the supply. For a second-price auction, the sealed-bid uniform-price auction is a poor extension as it is inefficient and leads to demand reduction (Ausubel and Cramton 1996), with bidders placing a lower bid on each additional as those bids may end up being the price. The Vickrey mechanism, on the other hand, maintains all of a second-price auction’s desirable properties such as efficiency and equilibrium bidding in dominant strategies. In our context, the Vickrey mechanism implies only that a syndicate will place an identical bid for each unit, and one syndicate’s price is determined by the other syndicate’s bid.

**Proposition 4.** Consider our benchmark model with \( n \geq 4 \) and an auctioneer selling \( k \leq n/2 \) identical assets.

1. In a multi-unit ascending (English) auction, an auction with two symmetric syndicates leads to lower expected revenue than an auction without syndicates.

2. There exists a \( r^*(n) \), \( \frac{1}{4} \leq r^*(n) \leq \frac{5}{12} \), such that, in a multi-unit sealed-bid (Vickrey) auction, an auction with two symmetric syndicates leads to higher expected revenue than an auction without syndicates if and only if \( k/n < r^*(n) \).

As in the single-unit case, syndication is never desirable when an open, ascending auction is used. Since bidders can already condition on the drop out points of bidders with lower signals, the price incorporates more than one bidder’s private information. Syndication does not produce sufficient enough advantages from information pooling to offset the loss of competition.

In the sealed bid case, the price reflects only the information of a single bidder. When bidders sufficiently outnumber the number of assets for sale, we recover our earlier results that syndication raises revenue. This suggests that our results on the competitive effect of syndicates are not unique to single-unit settings, but the multi-unit case requires some qualification, as syndicates lead to lower revenue when the number of assets is large. With one asset to sell, a second-price auction is used; the price reflects the information of the second-highest bidder, and thus reflects the second highest signal. With \( k \) assets, the \( k + 1^{th} \) bidder effectively sets the price. As \( k \) approaches \( n/2 \), the signal of the \( k + 1^{th} \) bidder approaches the median signal which is an unbiased estimate of the object’s true value. Thus, even though a single signal is reflected in the price, that signal becomes closer to the object’s true value. In this case, syndication again offers modest information pooling effects relative to the loss of competition.

As in the single-unit case, symmetry of syndicate sizes is not required for positive effects of syndicates to exist, as asymmetric cases may also lead to higher prices under syndication. In the multi-unit case, there are two bounds on the amount of asymmetry that may be permitted.
First, as in the one-unit case, too much asymmetry can reverse the result, making syndication anticompetitive in sealed bid auctions. Second, we require that the number of objects is not greater than the membership of the smaller syndicate. Overall, we find that that syndicated bidding in multi-unit environments (e.g., equity offerings or treasury securities) has similar competitive effects as in the single-unit case (e.g., take-overs).

6 Discussion

Past results suggest that when bidders have private values or common values with identically and independently distributed signals, the loss of competition implied by syndicate bidding reduces revenues. This is predicated on revenue equivalence which implies that all common auction formats are equally optimal. However, when bidders have a common value and their signals are correlated, as is likely to be the case in the book-building process of IPOs, auctions of treasury securities, bidding for takeover targets, and other financial markets, commonly used auction-like mechanisms are suboptimal.

The optimal revenue-maximizing mechanism is likely to be quite complicated. In practice, we observe much simpler selling formats, despite their suboptimality, as allocation mechanisms in financial markets. Among simple auctions, open English formats tend to outperform sealed-bid mechanisms (Milgrom and Weber 1982a). In open auction formats, bidders update their beliefs about the asset’s value each time another bidder drops out. This information exchange reduces bidders’ uncertainty and increases the resulting price. In sealed-bid auctions that do not allow bidders to update their estimates, syndication partially substitutes for the information loss by allowing groups of bidders to arrive collectively at better estimates. This comes at the cost of reducing the number of active bidders, exerting downward pressure on the price.

Our main results highlight another surprising effect of syndication that may improve the performance of sealed-bid auctions. For example, when signals are known to be upwardly biased or overly optimistic, the smallest signal is a reasonable representation of true value. In these cases, sealed-bid auction formats perform quite poorly in large markets but can be full-surplus extracting (and thus optimal) when only two syndicates exist.

This analysis suggests a screen for understanding the likely revenue effects of syndication in sealed-bid auctions predicated on a thought experiment. Imagine that a bidder receives several estimates of an asset’s value from his analysts. We write these signals on index cards and place them before the bidder in order from lowest to highest but face down, so that their exact values are not yet known. We ask the bidder to commit to a bidding function which depends on the order of these cards, but not their values. For example, a bidder who believes his analysts to be overly optimistic, often overstating the true value, may choose to bid based only on the smallest estimate. In another context, a bidder may elect to concentrate only on the highest estimates, or perhaps the average of all of them. This choice will, of course, depend
on the probability distribution of signals and the value, as well as the value function itself. Our result suggests that syndicates are likely to be revenue improving whenever sufficient weight is put on the lower values. Specifically, if the bidder has only two estimates available, syndicates can improve revenue as long as at least about $1/3$ of the weight is placed on the smaller of the two estimates, with no more than $2/3$ going to the higher estimate.

This paper focuses on the equilibrium effects of syndication on price. Where syndicates also serve a portfolio diversification role, for example, this would make them more competitive (or less anticompetitive). Conversely, syndication has been believed to facilitate coordination and collusion across syndicates, which would diminish from theoretical competitive gains, though recent empirical analysis calls this view into question (Boone and Mulherin 2008).

Several antitrust cases have recently challenged the way we analyze the competitive effects of American syndicates. Application of traditional antitrust paradigms and DOJ jurisdiction could thrust financial structures into an unfamiliar regulatory environment. Firms may be subject to arguments about theoretical missed gains, criminal statutes governing joint bidding, and heightened civil penalties as federal antitrust suits allow plaintiffs to recover threefold their damages. Yet, all of this rests on the assumption that joint bidding, absent capitalization or risk-sharing needs, skews auction prices in a socially undesirable direction. Our analysis suggests that this need not be the case.
References


Appendix: Proofs

Lemma 1. The distribution of $w$ conditional on having signal $s$ and on the highest signal among the remaining $n-1$ signals, $y_1$, being equal to $s$ is given by

$$f(w|s, y_1 = s) = \frac{(n-1)(s-w+\theta)^{n-2}}{(2\theta)^{n-1}}, \quad w \in [s-\theta, s+\theta].$$

Proof. First, note that

$$Pr\{y_1 = s|w, s\} = \frac{(n-1)(s-w+\theta)^{n-2}}{(2\theta)^{n-1}}$$

Then, whenever $|w-s| \leq \theta$, we have by Bayes’ Rule,

$$Pr\{W = w|s, y_1 = s\} = \frac{Pr\{y_1 = s|W = w, s\} Pr\{W = w|s\}}{\int_z Pr\{y_1 = s|W = z, s\} Pr\{W = z|s\} dz}$$

$$= \frac{(n-1)(s-w+\theta)^{n-2}}{\int_{s-\theta}^{s+\theta} (n-1)(s-z+\theta)^{n-2} dz}$$

$$= \frac{(n-1)(s-w+\theta)^{n-2}}{(2\theta)^{n-1}}$$

We will also make repeated use of the following identities:

$$\int_{s-\theta}^{s+\theta} f(w|s, y_1 = s) dw = 1$$

$$\int_{s-\theta}^{s+\theta} w f(w|s, y_1 = s) dw = s - \left(\frac{n-2}{n}\right) \theta$$

Proof of Claim. We prove a more general result that the given bidding function is the equilibrium for any value function given by:

$$v(w, s_1, \ldots, s_n) = \beta w + \left(1 - \beta\right) \frac{s_1 + s_n}{2}$$

where $\beta \in [0,1]$. When $\beta = 1$ this is the benchmark model, and when $\beta = 0$, this is the average of the maximum and minimum signals. We show that the equilibrium bid does not depend on $\beta$.

$$E[v|s, y_1 = s, w] = \beta w + \left(1 - \beta\right) \frac{1}{2} \left(s + E[y_{n-1}|y_1 = s, w]\right)$$

$$= \beta w + \frac{1-\beta}{2(n-1)} \left(ns + (n-2)(w - \theta)\right)$$

$$= \left(1 - \frac{n(1-\beta)}{2(n-1)}\right) w + \left(\frac{n(1-\beta)}{2(n-1)}\right) \left(s - \left(\frac{n-2}{n}\right) \theta\right)$$

$$= w - (1-\beta)\theta + \left(\frac{n(1-\beta)}{2(n-1)}\right) (s - w + \theta)$$
Using Eqs. (9) and (10), and Lemma 1, the equilibrium bid is given by

\[ b(s) = \int_{s-\theta}^{s+\theta} f(w|s, y_1 = s) E[v|s, y_1 = s, w] \, dw \]

\[ = s - \left( \frac{n-2}{n} \right) \theta - (1 - \beta)\theta + \int_{s-\theta}^{s+\theta} \frac{n(s-w+\theta)(n-1)}{(2\theta)^n} \, dw \]

\[ = s - \left( \frac{n-2}{n} \right) \theta \]

**Proof of Claim 2.** For part (ii), note that if \( n = 2 \), this is equivalent to the second-price auction. For \( n \geq 3 \), consider bidder 1 with signal \( s_1 \) and suppose other bidders follow the indicated strategy. Let \( s_{\text{min}} \) and \( s_{\text{max}} \) be the largest and smallest signals of the other bidders.

If \( s_1 > s_{\text{max}} \), bidder 1 wins by following the equilibrium strategy and receives a profit of \( E[v|s_1, s_{\text{min}}] - b(s_{\text{max}}) = \frac{1}{2}(s_1 - s_{\text{max}}) > 0 \). All bids that allow bidder 1 to win result in the same profit. If \( s_1 < s_{\text{max}} \), bidder 1 earns zero expected profits at any bid. Alternately, define \( y_i \) as the \( i \)th highest signal of bidder 1’s \( n-1 \) rivals.

\[ b(s|y_{n-1} = s_{\text{min}}) = \int_w E[v|w, s, y_1 = s, y_{n-1} = s_{\text{min}}] f(w|s, y_1 = s, y_{n-1} = s_{\text{min}}) \, dw \]

\[ = \int_{s-\theta}^{s+\theta} \frac{w}{2\theta - (s-s_{\text{min}})} \, dw \]

\[ = \frac{(s_{\text{min}} + \theta)^2 - (s - \theta)^2}{4\theta - 2(s-s_{\text{min}})} \]

\[ = \frac{s + s_n}{2} \]

**Claim 3’ (Generalization of Claim 3).** With two syndicates, if the value function depends only on \( w, s_{(1)} \), and \( s_{(n)} \), then the equilibrium bid is the expected value given one’s signals. In particular, if one syndicate has signals \( \{s_1, \ldots, s_m\} \), \( m \geq 1 \), the equilibrium bid is:

\[ \frac{1}{2} \min\{s_1, \ldots, s_m\} + \frac{1}{2} \max\{s_1, \ldots, s_m\} \]

for the benchmark case, and

\[ \alpha \min\{s_1, \ldots, s_m\} + (1 - \alpha) \max\{s_1, \ldots, s_m\} \]

for the general model.

**Proof.** Imagine a syndicate’s rival follows this strategy. There are four possibilities (parts in parentheses refer to the benchmark case):

1. **The rival has a higher min and higher max.** The object is (expected to be) worth less than the rival’s bid; thus any bid that assures losing this auction is a best reply.

2. **Rival has a lower max and a lower min.** The object is (expected to be) worth more than the rival’s bid; thus any bid that assures winning this auction is a best reply.

3. **Rival has a lower max and higher min.** Whenever the rival’s bid is lower than one’s own weighted max and min, the bidder wishes to win, but not otherwise.
4. **Rival has a higher max and lower min.** The rival is bidding the (expected value of the) object’s value. All bids are a best response.

The proposed bidding strategy satisfies all four conditions and, because of item 3, the proposed bidding strategy is a unique best response to a rival following the bidding strategy. □

**Proof of Propositions 2 and 3.**

**Syndicate Revenues:** Consider a bidder with \( m \) signals. We define

\[
\bar{\pi}_m = \frac{1}{2} \min\{s_1, \ldots, s_m\} + \frac{1}{2} \max\{s_1, \ldots, s_m\}
\]

as the average of the maximum and minimum of the bidder’s \( m \) signals. By Claim 3 with two syndicates, a syndicate with \( m \) signals bids \( \bar{\pi}_m \)

We want to calculate \( f_{\pi_m}(s|w) \), the distribution of \( \pi_m \) conditional on value. For \( m = 1 \), this is simply uniform on \([v - \theta, v + \theta]\). Suppose \( m \geq 2 \).

\[
f_{\pi_m}(s|w) = \int_{w-\theta}^{w+\theta} m(m-1)f(x)f(2\pi - x)|F(x) - F(2\pi - x)|^{m-2}dx
\]

\[
= \int_{\max\{w-\theta, 2s-\theta\}}^{\min\{w+\theta, 2s+\theta\}} \left( \frac{1}{4\theta^m} \right) m(m-1)|s - x|^{m-2}dx
\]

\[
= \begin{cases}
\int_{w-\theta}^{s} \frac{m(m-1)(s-x)^{m-2}}{2\theta^m} dx & s \leq w \\
\int_{s}^{2s-w-\theta} \frac{m(m-1)(s-x)^{m-2}}{2\theta^m} dx & s > w
\end{cases}
\]

\[
= \begin{cases}
\frac{-m(s-x)^{m-1}}{2\theta^m} & s \leq w \\
\frac{-m(s-x)^{m-1}}{2\theta^m} & s > w
\end{cases}
\]

\[
= \begin{cases}
\frac{m(s-w+\theta)^{m-1}}{2\theta^m} & s \leq w \\
\frac{m(w+\theta - s)^{m-1}}{2\theta^m} & s > w
\end{cases}
\]

And the CDF is given by:

\[
F_{\pi_m}(s|w) = \begin{cases}
\frac{1}{2} \left( \frac{s-w+\theta}{\theta} \right)^{m} & s \leq w \\
1 - \frac{1}{2} \left( \frac{w+\theta - s}{\theta} \right)^{m} & s > w
\end{cases}
\]

Let one syndicate have \( m \) signals, and the other have \( n - m \) signals, with \( 1 \leq m \leq n - 1 \). Revenue is equal to the expectation of \( \min\{\bar{\pi}_m, \bar{\pi}_{n-m}\} \), given by

\[
R^{2,m} = \int_{w-\theta}^{w+\theta} s \frac{m(s-w+\theta)^{m-1}}{2\theta^m} \left( 1 - \frac{1}{2} \left( \frac{s-w+\theta}{\theta} \right)^{n-m} \right) ds
\]

\[
+ \int_{w}^{w+\theta} s \frac{m(w+\theta - s)^{m-1}}{2\theta^m} \frac{1}{2} \left( \frac{w+\theta - s}{\theta} \right)^{n-m} ds
\]

\[
+ \int_{w-\theta}^{w} s \frac{(n-m)(s-w+\theta)^{n-m-1}}{2\theta^{n-m}} \left( 1 - \frac{1}{2} \left( \frac{s-w+\theta}{\theta} \right)^{m} \right) ds
\]

\[
+ \int_{w}^{w+\theta} s \frac{(n-m)(w+\theta - s)^{n-m-1}}{2\theta^{n-m}} \frac{1}{2} \left( \frac{w+\theta - s}{\theta} \right)^{m} ds
\]
\[
\begin{align*}
&= \int_{w-\theta}^{w} s \frac{m(s-w+\theta)^{m-1}}{2\theta^m} ds + \int_{w-\theta}^{w} s \frac{(n-m)(s-w+\theta)^{n-m-1}}{2\theta^{n-m}} ds \\
&\quad - \int_{w-\theta}^{w} s \frac{n(s-w+\theta)^{n-1}}{4\theta^n} ds + \int_{w}^{w+\theta} s \frac{n(\theta + s - s)^{n-1}}{4\theta^n} ds \\
&= w - \frac{\theta(n+2)}{2(m+1)(n-m+1)} + \frac{2(n+1)}{\theta} \\
&= w - \theta \left( \frac{(n+1)^2 - m(n-m)}{2(m+1)(n-m+1)(n+1)} \right)
\end{align*}
\] (35)

\[
\begin{align*}
&= \int_{w-\theta}^{w} s \frac{m(s-w+\theta)^{m-1}}{2\theta^m} ds + \int_{w-\theta}^{w} s \frac{(n-m)(s-w+\theta)^{n-m-1}}{2\theta^{n-m}} ds \\
&\quad - \int_{w-\theta}^{w} s \frac{n(s-w+\theta)^{n-1}}{4\theta^n} ds + \int_{w}^{w+\theta} s \frac{n(\theta + s - s)^{n-1}}{4\theta^n} ds \\
&= w - \frac{\theta(n+2)}{2(m+1)(n-m+1)} + \frac{2(n+1)}{\theta} \\
&= w - \theta \left( \frac{(n+1)^2 - m(n-m)}{2(m+1)(n-m+1)(n+1)} \right)
\end{align*}
\] (36)

No Syndicates Revenues (second-price): From Claim 1, the equilibrium bid is

\[
b(s) = s - \left( \frac{n-2}{n} \right) \theta
\]

Revenue is the above evaluated at the expectation of the second highest signal:

\[
R^{n,1} = \frac{n(n-1)}{(2\theta)^n} \int_{w-\theta}^{w+\theta} s(w + \theta - s)(s-w+\theta)^{n-2} ds - \left( \frac{n-2}{n} \right) \theta
\] (37)

\[
= w - \left( \frac{2(n-1)}{n(n+1)} \right) \theta
\] (38)

Effect of Syndicates (second-price): Syndicates raise revenue if \( R^{2,m} > R^{n,1} \):

\[
w - \theta \left( \frac{(n+1)^2 - m(n-m)}{2(m+1)(n-m+1)(n+1)} \right) \theta > w - \left( \frac{2(n-1)}{n(n+1)} \right) \theta
\] (39)

\[
4(m+1)(n-m+1)(n-1) > n(n+1)^2 - mn(n-m)
\] (40)

\[
m(n-m)(5n-4) > (n+1)(n^2 - 3n + 4)
\] (41)

Which is equivalent to

\[
\frac{1}{2} (1 - \phi) < \frac{m}{n} < \frac{1}{2} (1 + \phi)
\] (42)

where

\[
\phi = \sqrt{\frac{(n+4)(n^2 - 4)}{n^2(5n-4)}}
\] (43)

Because \( \phi \) is positive for \( n \geq 2 \), we obtain part (i) of Proposition 1. Differentiation reveals that \( \phi \) is decreasing with \( n \). Taking the limit of \( \phi \) as \( n \to \infty \) gives the bounds in Proposition 2.

No Syndicates Revenues (English): From the equilibrium bid in claim 2, revenue is the expectation of the average of the lowest and second-highest signal:

\[
R^{n,1} = \frac{n(n-1)(n-2)}{(2\theta)^n} \int_{w-\theta}^{w+\theta} \int_{s}^{w+\theta} \frac{1}{2}(s + s')(w + \theta - s')(s' - s)^{n-3} ds' ds
\] (44)

\[
= w - \frac{1}{n+1} \theta
\] (45)

Effect of Syndicates (English): For part (ii) of Proposition 1, we must show that
\[ R^{2n/2} < R^{n,1} \]
\[ w - \left( \frac{3n + 2}{2(n + 2)(n + 1)} \right) \theta < w - \frac{1}{n + 1} \theta \] (48)

which is equivalent to \( n > 2 \).

The proof of Claim 4 requires the following Lemma.

**Lemma 2.** Let the value function be given by

\[ v(w, s_1, \ldots, s_n) = \sum_{i=1}^{n} \gamma_i s_{(i)} \]

Where \( \gamma_i \geq 0 \), \( \sum \gamma_i = 1 \), and \( s_{(i)} \) is the \( i \)th highest signal from \( (s_1, \ldots, s_n) \). Define \( \gamma = \sum j \gamma_j \) as the “average” order statistic. The symmetric equilibrium bidding function without syndicates is:

\[ b(s) = s - \left( \frac{\gamma + \gamma_1 - 2}{n} \right) 2\theta \]

**Proof.**

\[ E[s_{(j)}|s, y_1 = s, w] = \begin{cases} \frac{s + i - 2}{n - j} & \text{if } j \leq 2 \\ \frac{s + i - 2}{n - j} & \text{if } j > 2 \end{cases} \] (49)

\[ E[v|s, y_1 = s, w] = \left( (\gamma_1 + \gamma_2)s + \sum_{j=3}^{n} \gamma_j \frac{n + 1 - j}{n - 1} s + \sum_{j=3}^{n} \gamma_j \frac{j - 2}{n - 1} (w - \theta) \right) \] (50)

\[ = \left( \frac{n + 1 - \gamma - \gamma_1}{n - 1} \right) s + \left( \frac{\gamma + \gamma_1 - 2}{n - 1} \right) (w - \theta) \] (51)

\[ b(s) = \int_{s - \theta}^{s + \theta} f(w|s, y_1 = s) E[v|s, y_1 = s, w] \, dw \] (52)

\[ = \int_{s - \theta}^{s + \theta} f(w|s, y_1 = s) \left( s - \left( \frac{\gamma + \gamma_1 - 2}{n - 1} \right) (s - w + \theta) \right) \, dw \] (53)

\[ = s - \left( \frac{\gamma + \gamma_1 - 2}{n} \right) 2\theta \] (54)

**Proof of Claim 4** By definition, \( 2 \leq \gamma + \gamma_1 \leq n \). Therefore, the bidding function in Lemma 2 satisfies:

\[ s \geq b(s) \geq s - \left( \frac{n - 2}{n} \right) 2\theta \] (55)

Consider the following value function:

\[ v(w, s_1, \ldots, s_n) = \alpha s_{(n)} + (1 - \alpha)s_{(1)} \]

The equilibrium bid is given by:

\[ b(s) = s - \left( \frac{n - 2}{n} \right) 2\alpha \theta \] (56)

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which, for $\alpha \in [0, 1]$, spans the range of possible bids.

**Proof of Proposition 3**

**Syndicate Revenues** A syndicate with $m$ signals, $\{s_1, \ldots, s_m\}$, bids:

$$\pi_m = \alpha \min \{s_1, \ldots, s_m\} + (1 - \alpha) \max \{s_1, \ldots, s_m\}$$

(57)

If $m = 1$, $b(s) = s$. For $m \geq 1$, following logic similar to the proof of Propositions 1 and 2

$$f_{\pi_m}(s|w) = \int_{w-\theta}^{s} m(m-1)f(x)f \left( \frac{s - \alpha x}{1 - \alpha} \right) \left( \frac{s - (1 - \alpha)x}{\alpha} \right) m^{-2} dx$$

(58)

$$+ \int_{s}^{w+\theta} m(m-1)f(x)f \left( \frac{s - (1 - \alpha)x}{\alpha} \right) \left( \frac{s - (1 - \alpha)x}{\alpha} \right) m^{-2} dx$$

(59)

$$= \left\{ \begin{array}{ll}
\int_{w-\theta}^{s} m(m-1) \left( \frac{1}{2\alpha} \right)^m \left( \frac{s-x}{1-\alpha} \right)^{m-2} dx \\
+ \int_{s}^{s-(\alpha(w-\theta))} m(m-1) \left( \frac{1}{2\alpha} \right)^m \left( \frac{s-x}{1-\alpha} \right)^{m-2} dx & s \leq w + (1 - 2\alpha)\theta \\
\int_{s}^{w+\theta} m(m-1) \left( \frac{1}{2\alpha} \right)^m \left( \frac{s-x}{1-\alpha} \right)^{m-2} dx & s > w + (1 - 2\alpha)\theta \\
\end{array} \right.$$ 

(60)

$$= \left\{ \begin{array}{ll}
m \left( \frac{1}{2\alpha} \right)^m \left( \frac{s-w+\theta}{1-\alpha} \right)^{m-1} & s \leq w + (1 - 2\alpha)\theta \\
m \left( \frac{1}{2\alpha} \right)^m \left( \frac{w+s+\theta}{2\alpha} \right)^{m-1} & s > w + (1 - 2\alpha)\theta \\
\end{array} \right.$$ 

(62)

And the CDF is given by:

$$F_{\pi_m}(s|w) = \left\{ \begin{array}{ll}
(1 - \alpha) \left( \frac{s-w+\theta}{2\alpha(1-\alpha)} \right)^m & s \leq w + (1 - 2\alpha)\theta \\
1 - \alpha \left( \frac{w+s+\theta}{2\alpha} \right)^m & s > w + (1 - 2\alpha)\theta \\
\end{array} \right.$$ 

(63)

With symmetric syndicates, both bidders have $m = n/2$. Revenue is given by

$$R_{n/2}^2 = \int_{w-\theta}^{w+\theta} 2sf_{\pi_{n/2}}(s|w)(1 - F_{\pi_{n/2}}(s|w))ds$$

(64)

$$= \int_{w-\theta}^{w+(1-2\alpha)\theta} s \left( \frac{s-w+\theta}{2\theta(1-\alpha)} \right)^{n/2-1} ds$$

(65)

$$- \int_{w-\theta}^{w+(1-2\alpha)\theta} s(1-\alpha) \left( \frac{s-w+\theta}{2\theta(1-\alpha)} \right)^{n-1} ds$$

(66)

$$+ \int_{w+(1-2\alpha)\theta}^{w+\theta} s\alpha \left( \frac{w-s+\theta}{2\theta(1-\alpha)} \right)^{n-1} ds$$

(67)

$$= w \left( \frac{n^2 - 3n - 2 - 2\alpha(n + \alpha)(n - 2)}{(n + 1)(n + 2)} \right) \theta$$

(68)
NO SYNDICATES REVENUES (second-price): The equilibrium bid is given by:

\[ b(s) = s - \left( \frac{n-2}{n} \right) 2\alpha \theta \]  
(69)

Revenue is the above evaluated at the expectation of the second highest signal:

\[ R_{n,1} = \frac{n(n-1)}{(2\theta)^n} \int_{w-\theta}^{w+\theta} s(w + \theta - s)(s - w + \theta)^{n-2} ds - \left( \frac{n-2}{n} \right) 2\alpha \theta \]  
(70)

\[ = w + \left( \frac{n(n-3)(1-\alpha) - (n^2 + n - 4)\alpha}{n(n+1)} \right) \theta \]  
(71)

EFFECT OF SYNDICATES: Syndicates raise revenue if

\[ R_{2,n}^{n/2} > R_{n,1} \]  
(72)

\[ \equiv \frac{(n^2 - 3n - 2) - 2\alpha(n + \alpha)(n - 2)}{(n+1)(n+2)} > \frac{n(n-3)(1-\alpha) - (n^2 + n - 4)\alpha}{n(n+1)} \]  
(73)

\[ \equiv (3n + 2)\alpha > n(1 + \alpha^2) \]  
(74)

which implies

\[ \alpha > \alpha^* \equiv \frac{(2 + 3n)2n - 1}{2n} - \frac{1}{2n} \sqrt{(n + 2)(5n + 2)} \]  
(75)

The derivative of \( \alpha^* \) with respect to \( n \) is:

\[ \frac{3n + 2 - \sqrt{(n + 2)(5n + 2)}}{n^2 \sqrt{(n + 2)(5n + 2)}} \]  
(76)

It can be confirmed that both the numerator and denominator are positive whenever \( n > 0 \). Thus, \( \alpha^* \) is increasing with \( n \). The limit of \( \alpha^* \) as \( n \to \infty \) is \( \frac{1}{2} \left( 3 - \sqrt{5} \right) \approx 0.382 \). When \( n = 4 \) (the lowest \( n \) so that symmetric mergers are possible), \( \alpha^* = \frac{1}{4} \left( 7 - \sqrt{33} \right) \approx 0.314 \). 

**Proof of Proposition 4**

NO SYNDICATES REVENUES: Following steps similar to Lemma 1, we derive the distribution of \( w \) conditional on the \( k^{th} \) highest signal among the remaining \( n-1 \) signals, \( y_k \), being equal to \( s \).

First, note that

\[ Pr\{y_k = s|w, s\} = \frac{(n-1)!(s-w+\theta)^{n-k-1}(w-s+\theta)^{k-1}}{(k-1)!(n-k-1)!(2\theta)^{n-1}} \]  
(77)

Then, by Bayes’ Rule,

\[ Pr\{W = w|s, y_k = s\} = \frac{Pr\{y_k = s|W = w, s\}Pr\{W = w|s\}}{\int_z Pr\{y_k = s|W = z, s\}Pr\{W = z|s\}dz} \]

\[ = \frac{(n-1)!(s-w+\theta)^{n-k-1}(w-s+\theta)^{k-1}}{(k-1)!(n-k-1)!(2\theta)^{n-1}} \]  
\( \square \)
Next, as in the proof of Claim 1, the equilibrium bid is given by

\[ b(s) = \int_{s-\theta}^{s+\theta} f(w|s, y_k = s)E[v|s, y_k = s, w] \, dw \] (78)

\[ b(s) = \int_{s-\theta}^{s+\theta} f(w|s, y_k = s) \, dw \] (79)

\[ = s - \left( \frac{n-2k}{n} \right) \theta \] (80)

Which corresponds to our single-unit equilibrium bid when \( k = 1 \). The resulting revenue from \( n \) bidders, 1 signal each, and \( k \) objects available, is the \( k+1^{th} \) highest bid times \( k \) total objects sold:

\[ R^{n,1,k} = k \int_{w-\theta}^{w+\theta} s \left( \frac{n! (1 - F(s|w))^k F(s|w)^{n-k-1} f(s|w)}{(n-k-1)!k!} \right) ds - k \left( \frac{n-2k}{n} \right) \theta \] (81)

\[ = k \int_{w-\theta}^{w+\theta} s \left( \frac{n! (w - s + \theta)^k (s - w + \theta)^{n-k-1}}{(n-k-1)!k!(2\theta)^k} \right) ds - k \left( \frac{n-2k}{n} \right) \theta \] (82)

\[ = kw + \left( \frac{n-2k-1}{n+1} \right) k\theta - \left( \frac{n-2k}{n} \right) k\theta \] (83)

\[ = kw - \left( \frac{2(n-k)}{n(n+1)} \right) k\theta \] (84)

**Syndicate Revenues:** Since the losing syndicate’s bids determine the price, each syndicate bids the same amount for every unit, and this bid is identical to the single-unit case. Thus, equilibrium revenues are simply \( k \) times the equilibrium revenues in Eq. (38) evaluated, for the symmetric case, at \( m = n/2 \):

\[ R^{2,n/2,k} = kw - \left( \frac{3n+2}{2(n+2)(n+1)} \right) k\theta \]

**Effect of Syndicates:** Syndicates raise revenue if

\[ R^{2,n/2,k} > R^{n,1,k} \] (85)

\[ \equiv \frac{2(n-k)}{n(n+1)} > \frac{3n+2}{2(n+2)(n+1)} \] (86)

\[ \equiv 4(n-k)(n+2) > (3n+2)n \] (87)

\[ \equiv \frac{k}{n} < r^*(n) \equiv \frac{(n+6)}{4(n+2)} \] (88)

By inspection, \( r^*(n) \) is decreasing in \( n \), with \( r^*(4) = \frac{5}{12} \) and \( \lim_{n \to \infty} r^*(n) = \frac{1}{4} \).