Lost Profits from Patent Infringement: The Simulation Approach

Gregory J. Werden*
Luke M. Froeb**
James Langenfeld***

Abstract
A patent owner is entitled to recover any additional profits that would have been earned but for infringement. This paper suggests the use of an adaptation of merger simulation to assess lost profits in patent infringement cases. A model of the industry with infringement is calibrated to observed prices and quantities and estimated demand elasticities. Lost profits are then estimated by calculating a new equilibrium without the infringing product(s).

JEL Classification Numbers: K29, L13, O34

Key Words: Simulation, Patents, Damages

Corresponding Author:
Gregory Werden
Suite 10,000
600 E Street NW
Washington, DC 20530
USA

---

* Economic Analysis Group, Antitrust Division, U.S. Department of Justice, Washington, DC 20530; gregory.werden@usdoj.gov. The views expressed herein are not purported to reflect those of the U.S. Department of Justice.

** Owen Graduate School of Management, Vanderbilt University; luke.froeb@vanderbilt.edu.

*** LECG/Navigant Consulting, Inc., Evanston, IL; James.Langenfeld@lecg.com.
Once a patent owner establishes infringement, the Patent Act requires the award of “damages adequate to compensate for the infringement.” Damages are assessed as lost profits, reasonable royalties, or a combination of the two. In both frequency and amount, awards of lost profits have become very significant in recent years (see Davis and Kedrowski, 1997). To recover lost profits for patent infringement, the patent owner must show a reasonable probability that it would have received the additional profits “but for” the infringement. The courts have prescribed no one particular method by which the patent owner must meet this burden. Precision in the estimation of damages is not required, and uncertainty tends to be resolved against those found to have infringed.

In considering whether to award lost profits, patent infringement cases devote a great deal of attention to an analog to antitrust market delineation. The issue is framed as whether there are “acceptable” non-infringing substitutes for the patentee’s product. Patent damage cases create the sort of false dichotomy that is familiar from antitrust cases, treating products determined to be “in the market” as almost perfect substitutes, and ignoring competition from outside.

If it is determined that there are no non-infringing substitutes for the patentee’s product, courts readily infer that the infringer diverted sales from the patentee and award lost profits for the sales diversion. The courts have sometimes noted that competition from an infringer affects prices, and rarely noted that price affects output. The courts do not appear to have brought together the sales diversion, price, and quantity issues in a unified analysis that reconstructs the market equilibrium but for the infringement.

We offer a practical way in which that reconstruction can be done. It is a straightforward adaptation of merger simulation, which has recently become conventional in antitrust. Merger simulation estimates the relevant demand relationships among competing products using price and quantity data, and calibrates a standard economic model of competitive interaction to observed prices and quantities. The calibrated model then predicts the effects of a merger on the prices and quantities of the merging firms and their rivals.

Simulating lost profits from patent infringement is similar to simulating the effects of a merger. Rather than extrapolate from the lower-price, pre-merger equilibrium to the higher-price, post-merger equilibrium, one extrapolates from the lower-price, with-infringement equilibrium to the higher-price, but-for-infringement equilibrium. While the former extrapolation involves internalizing competition among the merging products, the latter extrapolation involves eliminating the infringing product(s). Both simulations predict the effect of the change in structure on the sales and prices of competitors. And both simulations depend
on the structure of demand, which is partially estimated and partially assumed. Simulation eliminates the need to draw a bright line between close substitutes and not-so-close substitutes. The competitive effects of all substitutes are considered, in some cases by including the product in the simulation, and in others by allowing substitution to outside goods.

The courts’ focus on the infringer’s diversion of sales from the patentee has made them reluctant to award damages for lost profits when there are “acceptable” non-infringing substitutes, since the infringer’s sales were almost certainly not entirely diverted from the patentee. Lost profits damages have been awarded under the “market-share rule” when courts were satisfied that market shares provide a reasonable indication of how the infringer’s sales would be reallocated if it were eliminated from the market, i.e., when consumer preferences exhibit the Independence of Irrelevant Alternatives (IIA) property. Whenever it is reasonable to invoke the market-share rule, damages can be better assessed in a simulation based the highly parsimonious logit demand model, which exhibits the IIA property. We illustrate logit simulation with a hypothetical based on the facts of the case in which the market-share rule was established.

When the market-share rule and the logit model are not reasonable representations of actual demand conditions, simulations based on more flexible demand systems can be used. We illustrate this using nested logit demand in a hypothetical based on the facts of the leading case in which the courts declined to apply the market-share rule. Finally, we illustrate damage simulation when the patent owner and infringer compete in a procurement auction.

**Market Delineation in Patent Cases: Identifying “Acceptable” Substitutes**

The courts have viewed patent damages issues primarily through the analytic lens of causation, awarding the patentee lost profits associated with sales reasonably shown to have been diverted away by the infringement. The focus on causation derives from the fact that a patentee may lose sales and profits for many reasons other than infringement and should not be compensated by the infringer for such loses. The courts often emphasize that causation is a relatively simple matter with just two suppliers—the patentee and the infringer: “In a market with only two viable competitors, one may infer that the patentee would have made the infringer’s sales or charged higher prices but for the infringing competition.”

The two-supplier market is an abstraction; there are always substitutes for the patented product. Hence, the cases have asked whether available substitutes are “acceptable,” and “to prove that there are no acceptable noninfringing substitutes, the patent owner must show either that (1) the purchasers in the market place generally are willing to buy the patented product for
its advantages, or (2) the specific purchasers of the infringing product purchased on that basis.\textsuperscript{vii} Often, only nearly identical products are deemed “acceptable.” In one oft-cited case, only wheelbarrows with the particular design features of the patentee’s product were deemed “acceptable substitutes.”\textsuperscript{viii}

The definition of “acceptable substitutes” is analogous to market delineation in antitrust cases. In one leading case, the court even framed the issue as whether “the patent owner and the infringer compete in the same market.”\textsuperscript{ix} As a general rule, “the market in patent cases is defined in very narrow terms.”\textsuperscript{x} Culbertson and Weinstein (1988), Gould and Langenfeld (1997, pp. 465–66, 474-76), Jarosz and Page (1993, pp. 316–19), Krosin and Kozlowski (1990, pp. 70–72), and Meyer (1991, pp. 1372, 1388–91) all have questioned the narrowness of patent markets and suggested the importation of antitrust market delineation from the Merger Guidelines or the antitrust case law.\textsuperscript{xi} While that would be a substantial improvement, there would remain problems well known in merger analysis.\textsuperscript{xii}

The purpose of market delineation is to simplify a complex world, by dividing substitutes into two groups—one in which all products generally are treated as equally good substitutes for each other, and a second consisting of products usually considered irrelevant. The treatment of products in both groups typically is a rough approximation of the real world and often ignores much that is significant. Moreover, so much is likely to be at stake in the line drawing process, that the focus of litigation gets diverted from the issues of ultimate consequence to the line drawing itself. These problems have been noted in antitrust (see Werden, 1997a, pp. 367–71), and merger simulation was developed to avoid arbitrary line drawing and refocus attention on the issues that matter.

The Components of Lost Profits Damages

Even when market delineation is trivial because there clearly are no “acceptable” non-infringing substitutes for the patentee’s product, several causation issues may remain. As the courts have recognized, the patentee’s lost sales may be less than the infringer’s sales because the products are differentiated in some important way. More importantly, one would always expect some price effect from competition between the patentee and the infringer, since duopoly nearly always results in a lower price and higher market quantity than monopoly. Hence, the effect of infringement would never be merely to divert away some of the patentee’s sales. The proper assessment of lost profits resulting from infringement should focus not on the infringer’s sales, but rather on the price and quantity that would have prevailed but for infringement.
The case law refers to the price effect of infringement as “price erosion.” Damages from price erosion are awarded in only a small minority of cases (see Davis and Kedrowski, 1997, pp. 13–14), even though price erosion (to some extent) must be a nearly universal phenomenon. Moreover, a lower price implies a higher quantity, other things being equal, so a necessary consequence of price erosion is “quantity accretion.” As obvious as this point is to economists, it has been recognized by very few courts. A proper assessment of “damages adequate to compensate for the infringement” must consider sales diversion, price erosion, and quantity accretion, and since they are interrelated, all three must be considered in conjunction.

The failure to account for price erosion surely can be significant. Rampant infringement could drive the patentee’s price down to its marginal cost, in which case lost profits damages based solely on sales diversion would be zero. Price erosion also can be modest, in which case, damages based solely on sales diversion would be a good approximation of the true lost profits. To get some perspective on the likely significance of price erosion, we borrow an example from Werden, Beavers, and Froeb (1999), who also present a graphical analysis. Consider a homogeneous product industry with linear demand and constant marginal cost, and assume that infringement results in a symmetric Cournot duopoly. It is easily shown that the patentee’s actual lost profits are 20% greater than damages calculated on the basis of sales diversion alone. On the other hand, accounting for price erosion but not quantity accretion, overestimates damages by 60%.

The impacts of price erosion and quantity accretion are less likely to be significant when products are substantially differentiated. The less close a substitute the infringer’s product is for the patentee’s product, the less is the effect of the infringement on the patentee’s sales and price. If the patentee’s and infringer’s products are distant substitutes, the infringement may result in very little lost profit. There is also another important contrast between homogeneous and differentiated products industries. With homogeneous products, the infringer competes down industry price, causing a movement down the demand curve. With differentiated products, there is a “variety enhancement” effect; by increasing product variety, the infringer effectively shifts the industry demand curve out. With homogeneous products, the patentee’s lost profits are necessarily greater than the infringer’s profits (assuming the two have the same costs) because of the price and quantity effects, but with differentiated products, the patentee’s lost profits may be significantly less than the infringer’s profits.

**Calculating Lost Profits in the Absence of “Acceptable” Non-Infringing Substitutes**

If the infringer is the patentee’s only important competitor, the patentee would be a
monopolist without the infringement (at least with respect to a residual demand curve). Thus, the patentee would maximize profits by setting price or quantity to equate its price-cost margin with the reciprocal of its demand elasticity. The patentee’s elasticity of demand at the monopoly price cannot be measured directly; rather, it must be determined through extrapolation of the demand curve from the price and quantity with infringement to the price and quantity without infringement. This can be done by first estimating the elasticity of demand for the products of the patentee and infringer together.\textsuperscript{xiv} The estimation yields a value for the elasticity valid with the infringement, and an assumption must be made about the shape of the demand curve to extrapolate to the monopoly price.\textsuperscript{xv}

There are two approaches to identifying the patentee’s marginal cost. Patent damages cases have long relied on accounting data to estimate the patentee’s marginal cost in order to determine the lost profit associated with sales diversion, and it is possible to infer marginal costs from a model of the industry with the infringement.\textsuperscript{xvi} To do the latter, relevant demand parameters are first estimated and those estimates are combined with data on observed prices and quantities to calibrate a demand system for patentee and the infringer. The patentee’s and infringer’s first-order conditions for profit maximization are then solved for the marginal costs that yield as an equilibrium the prices and quantities observed with infringement.

Both of the foregoing approaches may be unworkable unless the patentee’s marginal cost is roughly constant in the relevant range,\textsuperscript{xvii} but that is likely to be the case unless there are significant capacity constraints. Patent infringement cases already consider whether the patentee has the capacity to sell the infringer’s quantity in determining whether to award lost profits rather than reasonable royalties.

Treatment of the patentee as a monopolist does not mean that this analysis is valid only when the patentee faces no competition other than the infringer. The key assumption on which the analysis is predicated is not that there are no non-infringing substitutes for the patentee product, but rather that the patentee’s pricing does not affect pricing strategies for any non-infringing substitutes. Hence, this approach is applicable when the prices of non-infringing substitutes are determined by supply and demand forces independent of the patentee’s price and when the non-infringing substitutes form a competitive fringe.

The Market-Share Rule

The courts have been wary about awarding lost profits when there are non-infringing substitutes because of difficulties in inferring causation, but some cases have made that inference. Lost profits have been awarded if non-infringing substitutes were considered
insignificant or if the courts were satisfied that the patentee’s market share was a reasonable basis for determining the extent to which the infringer diverted sales from the patentee rather than non-infringing substitutes.

The “market-share rule” was established in *State Industries, Inc. v. Mor-Flo Industries, Inc.*, xviii in which State was awarded lost profits for Mor-Flo’s infringement, even though the patentee and infringer were not the only producers of “acceptable” substitutes. Mor-Flo infringed State’s patent on a method of insulating water heaters with foam, and foam insulated water heaters are more energy efficient than those insulated with fiberglass. Finding that State’s nationwide share of energy-efficient, residential, gas water heaters was 40% during the period of infringement, xix the courts awarded lost profits damages on 40% of Mor-Flo’s sales, with a reasonable royalty awarded on the remainder.

We understand the courts to have found that, if Mor-Flo’s infringing product were eliminated from the market, its sales would have been reallocated among the patentee and sellers of non-infringing substitutes in proportion to those firms’ market shares. But as noted by Jarosz and Page (1993, p. 318 n.22) and Krosin and Kozlowski (1990, p. 82), this finding supports a calculation somewhat different from that actually performed in *Mor-Flo* and other cases. The court in *Mor-Flo* multiplied the infringer’s quantity by the State’s share of total market quantity to arrive at an estimate of the sales diversion from State to Mor-Flo. It would have made more sense, however, to multiply Mor-Flo’s by the States’s share of market quantity excluding that sold by Mor-Flo. Mor-Flo’s share was about 20% according to data in the district court’s opinion. Assuming the figure was exactly 20%, State should have been awarded lost profits on 50%, rather than 40%, of Mor-Flo’s sales. We assume hereafter that the “market-share rule” is not the calculation actually performed in *Mor-Flo*, but rather the one we find more sensible.

The finding we attribute to the court in *Mor-Flo* is a way of stating the IIA property. The conventional, verbal statement of the IIA property is that the odds ratio of any two choices is independent of the other possible choices. The IIA property probably never holds exactly in a real-world industry, but it may be a reasonable approximation in many.

**Simulating Lost Profits with Non-Infringing Substitutes: The Logit Model**

The IIA property is associated in economics with the logit model, which was originally developed using the IIA as a starting point (see Werden, Froeb, and Tardiff, 1996, pp. 86–87). With the logit model choice probabilities for individual products $i$ take the form:
\[ \pi_i = \frac{\exp(\alpha_i - \beta p_i)}{\sum_j \exp(\alpha_j - \beta p_j)} \]

where the \( \alpha_i \) are choice-specific constants summarizing product preferences, the \( p_i \) are the prices of the product, and \( \beta \) is a demand parameter related to elasticity. The own and cross elasticities of demand are easily shown to be

\[ \varepsilon_{ii} = -\beta p_i (1 - \pi_i) \]

\[ \varepsilon_{ij} = \beta p_j \pi_j. \]

When using the logit model in simulations, it is convenient to group all the relatively unimportant products into a single “outside good” with choice probability \( \pi_o \) and a price that is assumed to be constant (and hence can be set equal to zero). Rather than have \( \pi_o \) reflect the actual likelihood of the choice of an outside good, we find it convenient to use it to scale the choice probabilities for the “inside goods” (i.e., those other than the outside good). This is done by adding a second demand parameter \( \varepsilon \), which is the aggregate elasticity of demand for the inside goods. There are two intuitive ways of defining this elasticity, and for both,

\[ \varepsilon \equiv -\beta \bar{p} \pi_o. \]

The aggregate elasticity can be defined such that the price changes for the inside goods are equiproportional, in which case \( \bar{p} \) is defined as the share-weighted average price for the inside goods. The aggregate elasticity also can be defined such that the price changes for the inside goods are equal in absolute terms, in which case \( \bar{p} \) is defined as the unweighted average price for the inside goods.

When courts find that the IIA property is a reasonable approximation, it is straightforward to use the logit model to estimate lost profits, accounting for not just sales diversion, but also price erosion and quantity accretion. Normally, that would be done by first estimating the two demand parameters with data on actual choices or possibly survey data. The estimated demand parameters along with shares and prices for the inside goods are sufficient to characterize an industry. The simulation proceeds by solving the first-order conditions that define a Bertrand equilibrium with the infringement for the implied marginal costs. The simulation is completed by solving the first-order conditions without the infringement for the Bertrand equilibrium absent the infringement.

The published court opinions in Mor-Flo provide neither estimated demand elasticities nor
data from which to estimate them, but the district court found facts about State’s sales and profits that indicate that its average price-cost margin during the infringement period was 0.1635. This margin can serve in place of one of the two demand parameters. For any given value of $\varepsilon$, there is a unique $\beta$ that implies the price-cost margin found by the court (given the share and price assumptions immediately below). We consider three different values of $\varepsilon$ (–0.5, –1, and –2) and the corresponding values of $\beta$ (11.948, 9.484, and 8.7746) that imply the price-cost margin found by the court. Because less energy efficient water heaters and electric powered water heaters were available, there are strong reasons to suspect that the demand for energy-efficient, residential gas water heaters was elastic, and therefore that the most relevant of the three elasticities of demand probably is –2.

The court opinions indicate that there were five firms in the industry and that State had a 40% share. Lacking information on the remaining shares, we assume they were equal. Financial data in the opinion indicate that the patentee’s price was somewhat greater than that of the infringer, and we normalize the patentee’s price to 1 and rival’s prices to 0.9.

Since we have normalized both price and output (by using shares), it is useful to consider the calculated damages relative to some convenient benchmark, and we use State’s profits with infringement. The table below presents calculated damages assuming just a single rival was infringing. It also presents an estimate of damages based the application of the market-share rule. In general, the market-share rule can over- or understate actual damages, and that is also the result here, depending on the aggregate elasticity of demand. We believe that the most elastic demand assumption is the most realistic, and for that assumption, the market-share rule overstates the simulated damages by 28%.

**First Table**

Even when the logic of the market-share rule is correct, as it is by construction with logit demand, the effects on the patentee’s profits of price erosion, quantity accretion, and variety enhancement can be significant. With differentiated products, failing to account for the variety enhancement effect easily can be the greatest source of error. If it is possible to estimate the relevant demand parameters or even make educated guesses about them, simulation is likely to offer an improvement over the application of the market-share rule.

**Generalizing the Market-Share Rule**

The IIA assumption implicit in the market share rule is likely to be an inadequate description of consumer preferences in many particular cases. When the IIA property is not a
reasonable characterization of preferences, the sort of damage simulation described in the previous section can be undertaken using a somewhat more flexible demand system. In merger simulation, four demand systems have been used—linear, isoelastic, AIDS, and nested logit. Other demand systems that could be used are the mixed logit system (Berry, Levinsohn and Pakes, 1995; Brownstone and Train, 1999). We illustrate only the nested logit model.

The leading case declining to apply the market-share rule is *BIC Leisure Products, Inc. v. Windsurfing International, Inc.* Windsurfing held a patent on sailboards that perhaps encompassed all boards in the market. The patent was licensed to many competitors, generally at a 7.5% royalty. In addition, Windsurfing manufactured sailboards that embodied its patent. The district court held that it was reasonable to assume that, had BIC not been in the market, its sales would have been distributed among the other competitors in proportion to their market shares; hence, the district court awarded lost profits by applying the market-share rule. The court of appeals reversed, holding that Windsurfing did not establish causation, in light of substantial differences in product design and price. Damages were limited to reasonable royalties—the 7.5% Windsurfing was charging others.

The court of appeals probably was right to reject the IIA assumption in this case and to reverse the award premised on the market-share rule. Rejecting the IIA assumption, however, should not mean that damages are limited to reasonable royalties. To the extent the court found that causation was unproved, we think the court erred by failing to focus on whether the infringement caused Windsurfing to lose profits by selling less or at a lower price. To the extent the court simply could not fathom how to assess lost profits, we offer simulation a method for doing so.

Because the published opinions contain neither demand elasticity estimates nor data from with we can estimate the elasticities, we are unable to simulate damages in this case in quite the way described above. We must make some simplifying assumptions, including a parsimonious specification for demand; hence, we assume a nested logit model in which all of the nests have the same nest parameter, $\delta$. With $S$ nests and the set of products contained in nest $s$ denoted $N_s$, the probability of choice $i$ in nest $j$ is

$$
\text{Prob}(i|j) = \frac{\exp(\delta x_{ij})}{\sum_{k \in N_j} \exp(\delta x_{kj})}
$$

We divide the various competing products into four nests on the basis of their prices. We do not know from the published court opinions how strong the nests should be, and arbitrarily assume a nest parameter of 0.8. This makes the nests strong enough to matter while allowing significant competition with products outside a nest. We assume that the aggregate
elasticity of demand for all sailboards is \(-1\) in the with-infringement equilibrium. Finally, we calibrate the logit \(\beta\) to imply a marginal cost for the patentee equal to that found by the district court. In doing this calibration and in computing the damages, we account for the fact that the patentee was earning a 7.5% royalty on licensed sales by the non-infringing rivals.

The table below presents four damage estimates: the original award made by the district court based on the market-share rule, our simulation estimate, a second simulation estimate imposing the IIA assumption (i.e., with a nest parameter of 1), and the royalty-only award made by the court of appeals. The damages are separated into components relating to the patentee’s own sales and the sales of licensees, although the royalty-only award fits neither category. Perhaps most significantly, the original award, based on the market-share rule, is nearly double the simulation estimate imposing the IIA assumption. Simulation results in a lower damage estimate than the market-share rule because simulation properly accounts for quantity accretion and variety enhancement.

Second Table
Lost Profits with Non-Infringing Substitutes: Auctions

Although differentiated consumer products were involved in leading cases on lost profits from patent infringement, the simulation approach has more general application. Simulation can be usefully employed to assess lost profits from infringement whenever the relevant competitive interaction can be modeled well using a static oligopoly model with a unique equilibrium. As a final illustration of the potential applications of damage simulation, we posit that the patentee, the infringer, and others compete to sell through an auction mechanism. For simplicity, we consider a private-values, second-price auction—-a procurement auction that awards a single contract to the low bidder.

In a second-price auction (with a purchasing auctioneer), the low bidder wins the auction and sells at a price equal to the second-lowest bid. As explained by Vickrey (1961), the each bidder’s dominant strategy is to bid its cost, because one’s own bid determines whether that one wins the auction but does not affect the price paid. In the private-values context, a second-price auction is equivalent to an oral auction.

The patentee is damaged by the infringement in any procurement in which the patentee and the infringer are the two lowest bidders. In such instances, there is necessarily price erosion and possibly sales diversion as well. There is sales diversion if the infringer is the low bidder and the patentee is the next-lowest bidder. In that event, the patentee would have won the auction but for the infringement. There is price erosion in that event as well, because the infringer’s low bid causes the price-determining, second-lowest bid to be the patentee’s bid instead of the bid just above it. There is also price erosion if the patentee is the low bidder and the infringer is the next-lowest bidder. In that event, the infringer’s bid causes the price-determining, second-lowest bid to be the infringer’s bid instead of the bid just above it. Computing damages, thus, involves a determination of how frequently the patentee and infringer are the two lowest bidders and what the price would have been absent the infringer’s bid.

Computing damages as actual lost profits is simple with complete data on each auction, including losing bids. Such data, however, often are unavailable, since only winning bids may be recorded. Data on just winning bids is sufficient to compute damages based on the patentee’s simulated expected lost profits. To do this, one first calibrates a tractable model to the with-infringement equilibrium, then determines the expected equilibrium after eliminating the infringer as a bidder. Froeb, Tschantz, and Crooke (1999a, 1999b) have identified a class of auction models that can be characterized by market shares (how frequently each bidder wins) and the distribution of the minimum cost (over all bidders). Following Wachrer and Perry
(1999), they assume that bidders take independent cost draws from cost distributions that differ across bidders but are all “power-related.” What this means is that bidder $i$’s cumulative cost distribution, $1 - G_i(x)$, is related to some parent distribution, $F(x)$, in the following manner:

$$G_i(x) = \left[1 - F(x)\right]^r_i.$$ 

This form of cost asymmetry would result if bidders drew from the same cost distribution but differed in the number of independent draws, $r_i$, they took from it. The greater $r_i$, the lower bidder $i$’s expected cost and the greater its expected share of wins.

If $s_i$ is the probability that bidder $i$ wins the auction or the “share” of bidder $i$, and $1 - G_{\text{min}}(x)$ is the cumulative distribution of minimum cost over all bidders, the family of distributions can be characterized as follows:

$$G_i(x) = \left[G_{\text{min}}(x)\right]^{r_i}, \quad G_{\text{min}}(x) = \left[1 - F(x)\right]^{r_{\text{min}}}, \quad r_{\text{min}} = \sum_{i=1}^{n} r_i, \quad s_i = r_i / r_{\text{min}}.$$ 

And letting

$$r_{-i} = r_{\text{min}} - r_i, \quad G_{-i}(x) = \left[1 - F(x)\right]^{r_{-i}},$$

the closed-form expression for the distribution of the winning price (which is the second-lowest bid), $p_i$, conditional on bidder $i$ having the lowest cost draw is

$$F_{p_i}(x) = \frac{1}{s_i} \left[1 - G_{-i}(x)\right] + \frac{1}{s_i} \left[1 - G_{i}(x)\right].$$

The expected value of bidder $i$’s winning bid is

$$E(p_i) = \frac{1}{s_i} \mu(r_{-i}) + \left[1 - \frac{1}{s_i}\right] \mu(r_{\text{min}}),$$

where $\mu(r)$ is the mean of a random variable with cumulative distribution $1 - [1 - F(x)]^r$. Both expressions are weighted averages, with weights based on the bidder’s probability of winning, $s_i$. The quantities averaged are associated with the minimum cost for bidders other than $i$ and for all bidders. Expected bids are decreasing in the respective shares because larger firms expect to win at higher prices than smaller firms. Losing bidders set the price, and low-cost, large-share bidders expect to win while bidding less aggressively than higher-cost rivals.

For a simple illustrative simulation, we assume that the distribution of minimum cost has an extreme-value type III distribution. This distribution gives rise to logit demand (Werden and Froeb, 1994) and logit auction models (Froeb, Tschantz, and Crooke, 1999a, 1999b) and is relatively easy to estimate (Brannman and Froeb, forthcoming). We assume that the cost
distribution for the minimum cost over all bidders has a mean of 100 and a standard deviation of 10. The expected cost of the winner (which has the minimum cost), thus, is also 100. We assume that there are three bidders with infringement—the patentee, the infringer, and one non-infringing competitor—with shares as indicated in the table below. The table also displays the shares, expected winning bids, and profits of the three bidders, both with and without infringement.xxvii

The infringer’s share is redistributed to the remaining bidders in proportion to their shares with infringement. This again is the IIA property, which underlays the courts’ market-share rule. The IIA property in this model stems from the assumption of power-related cost distributions, and it can arise with some other assumptions as well. The IIA property does not hold if there is correlation among some bidder’s cost draws (see Bajari, 1996). One way to introduce such correlation is through the addition of one or more nests. Introducing a positive correlation in two bidders’s cost draws increases the frequency with which they are the two lowest bidders. A positive correlation between the costs of the patentee and infringer increases expected lost profits.

Third Table

Conclusion

We offer simulation as a methodology for assessing lost profits damages in patent damages cases. Simulation provides reasonable lost profits estimates in cases with and without non-infringing substitutes, and accounts for potentially significant effects the courts have often ignored. Simulation accounts for price erosion and the corresponding quantity accretion, and it accounts for the effects infringement can have on total market quantity as a result of enhancing product variety.

Simulation should help to refocus the causation analysis in patent infringement cases. When an infringer competes with the patentee, that competition results in lost profits for the patentee, and proof of significant competition ought to be sufficient to establish the causation necessary for recovery of lost profits. What remains is for the patentee to make a reasonable estimate of the lost profits, and simulation provides a methodology for doing so.

Simulation, of course, can provide only a reasonable estimate. There will always be a potential for error due to the fact that the assumptions on which the simulations are based are never precisely right, and there is statistical variation in any estimation of demand elasticities. But the tendency of the courts to resolve uncertainty against the infringer should assure that
simulated damage estimates are not considered speculative. Moreover, nothing in the case law prohibits refinements in methods of assessing damages.
## Damages as a Percentage of Profits with Infringement in a Hypothetical Case Based on *Mor-Flo*

<table>
<thead>
<tr>
<th>Method of Computation</th>
<th>Elasticity of Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-0.5</td>
</tr>
<tr>
<td>Simulation</td>
<td>19.7</td>
</tr>
<tr>
<td>Market-Share Rule</td>
<td>17.7</td>
</tr>
</tbody>
</table>
## Court and Simulation Damages Estimates in *BIC Leisure*

(Thousand dollars)

<table>
<thead>
<tr>
<th></th>
<th>Licensee Sales</th>
<th>Patentee Sales</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original Award</td>
<td>750</td>
<td>2,102</td>
<td>2,852</td>
</tr>
<tr>
<td>Simulated Damages, $\delta = 1$</td>
<td>376</td>
<td>1,115</td>
<td>1,491</td>
</tr>
<tr>
<td>Simulated Damages, $\delta = .8$</td>
<td>403</td>
<td>890</td>
<td>1,293</td>
</tr>
<tr>
<td>Royalty Only Award</td>
<td></td>
<td></td>
<td>1,079</td>
</tr>
</tbody>
</table>
## Simulated Damages in a Hypothetical Auction Setting

<table>
<thead>
<tr>
<th></th>
<th>Patentee</th>
<th>Infringer</th>
<th>Third Bidder</th>
</tr>
</thead>
<tbody>
<tr>
<td>With Infringement</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>expected share</td>
<td>0.3</td>
<td>0.5</td>
<td>0.2</td>
</tr>
<tr>
<td>expected winning bid</td>
<td>109.3</td>
<td>110.8</td>
<td>108.7</td>
</tr>
<tr>
<td>expected profit</td>
<td>2.8</td>
<td>5.4</td>
<td>1.7</td>
</tr>
<tr>
<td>But for Infringement</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>expected share</td>
<td>0.6</td>
<td></td>
<td>0.4</td>
</tr>
<tr>
<td>expected winning bid</td>
<td>117.3</td>
<td></td>
<td>115.4</td>
</tr>
<tr>
<td>expected profit</td>
<td>7.1</td>
<td></td>
<td>4.0</td>
</tr>
</tbody>
</table>
References


Waechrer, Keith and Perry, Martin K., “The Effects of Mergers in Open Auction Markets,” BLS working paper 322, June 1999


Notes

i. 35 U.S.C. § 284.


iii. Our proposal is not entirely novel. Authors Froeb and Langenfeld used a version of it in consulting work on an actual patent infringement case, as did the opposing expert, David Evans. An exercise based on that case is available on-line at http://www.antitrust.org/simulation.html. The work of many other economists on patent damages cases accounts for important economic forces implicit in our suggested analysis.

iv. For discussions of the rationale for, and details of, merger simulation, see Crooke, Froeb, Tschantz, and Werden (1999), Froeb and Werden (1996), and Werden (1997a, 1997b, 1997c).

v. Werden, Froeb, and Beavers (1999) discuss the case law in more detail.


viii. Radio Steel & Manufacturing Co. v. MTD Products, Inc., 788 F.2d 1554, 229 U.S.P.Q. (BNA) 431 (Fed. Cir. 1986). The features cited were the ability to be shipped unassembled and the absence of a brace along the rear of the legs.


xii. The importation of antitrust market delineation, however, should produce satisfactory results when damage simulation is impractical.


xiv. Demand estimation is common in antitrust cases and has been done in some infringement cases. The sort of price variation that is essential in accurately estimating demand elasticities may be more likely to exist in the infringement cases because it results from the infringement itself. Simulation can also be performed using elasticities inferred from various economic data or intuited from non-quantitative evidence.
xv. As indicated by research on merger simulations (Crooke, Froeb, Tschantz, and Werden, 1999),
damages may be substantially greater if the demand curve is assumed to be isoelastic or AIDS than if it is assumed to be linear or logit.

xvi. When marginal costs are inferred, accounting estimates are useful cross checks.

xvii. It may be possible to estimate marginal cost functions when marginal costs are not constant.


xix. Mor-Flo argued that State’s national market share was irrelevant, since Mor-Flo sold mostly in California, where State had a very small share. The courts rejected this argument on a rather unclear basis, but it seems likely that Mor-Flo had a valid point. Water heaters are bulky and relatively expensive to transport. Consequently, manufacturers tend to have higher shares of sales nearer to their manufacturing plants. Mor-Flo had a plant in California, while State did not have a plant anywhere in the western United States. On the other hand, State and Mor-Flo both had plants in Tennessee.

xx. Shortly after the period of the alleged infringement, the Department of Justice challenged the merger of two firms in the industry. The relevant market alleged in the complaint was all residential water heaters, suggesting that neither energy-efficient water heaters nor gas water heaters were a distinct relevant market for antitrust purposes.

xxi. These are discussed and compared by Crooke, Froeb, Tschantz, and Werden (1999). Merger simulation with linear and isoelastic demand is discussed by Werden (1997c), and merger simulation with AIDS demand is discussed by Hausman and Leonard (1997).


xxiii. Additional details on the assumptions of this simulation and its results are provided by Werden, Froeb, and Beavers (1999).

xxiv. As stated at the outset, damages simulation is an adaptation of merger simulation. For discussions about merger simulation in first-price or sealed bid auctions, see Tschantz, Crooke, and Froeb (1997) and Dalkir, Logan and Masson (forthcoming).

xxv. We assume that the auctioneer does not set a reserve price and the contract is always let. Relaxing this assumption gives the auctioneer some bargaining power. It is a straightforward in that case to compute optimal reserve prices.

xxvi. For high-bid auctions, this class of distributions can be characterized in terms of the shares and the distribution of the maximum value.

xxvii. Eliminating the infringer changes the distribution of the minimum cost over all bidders, increasing its mean by 5.4.