Unilateral Competitive Effects of Horizontal Mergers: Theory and Application Through Merger Simulation

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Introduction

Horizontal mergers—those of direct competitors—give rise to unilateral anticompetitive effects if they cause the merged firm to charge a higher price, produce a lower output, or otherwise act less intensely competitive than the merging firms, even though non-merging rivals do not alter their strategies. Unilateral effects contrast with coordinated effects arising if a merger induces rivals to alter their strategies, resulting in some form of coordination or reinforcement of ongoing coordination. The term “unilateral” is used because the merged firm pursues its unilateral self-interest.

Unilateral merger effects flow from the internalization of the competition between the merging firms. The simplest case is the merger of duopolists when the pre-merger oligopoly game yields a unique equilibrium more competitive in any relevant sense than the monopoly equilibrium. Through the merged firm’s pursuit

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of self-interest, the merger causes a shift from the duopoly equilibrium to the less-competitive monopoly equilibrium. While merger to monopoly is the simplest example of unilateral effects, it may be the least common and least interesting, so the remainder of this paper considers mergers not resulting in monopoly.

Unilateral effects of mergers arise in one-shot oligopoly games with Nash, non-cooperative equilibria, including the classic models of Cournot (1838), Bertrand (1888), and Forchheimer (1908) (dominant firm). In Bertrand oligopoly, for example, a merger combining two competing brands of a differentiated consumer product, and not reducing costs, necessarily leads to unilateral price increases, even if only very small price increases. The merged firm accounts for the increase in sales of either of the two brands resulting from an increase in the price of the other, and therefore finds it in its unilateral self-interest to raise the prices of both. Pursuing the unchanging strategies optimal in the Bertrand model, firms selling competing brands respond by raising their prices (unless their demands are unaffected by the merged firm’s price increases). The post-merger equilibrium reflects the merged firm’s response to the responses of non-merging firms, the non-merging firms’ responses to those responses, and so forth.

Although unilateral effects theories are based on ideas that are quite old as economic theory goes, explicit application of these ideas to merger policy was quite limited prior to the release of the *Horizontal Merger Guidelines* (1992). One reason unilateral effects theories became prominent so recently is that most economists paid little attention to the Cournot and Bertrand models during the formative era for merger enforcement policy in the United States—from the late 1940s through the late 1960s. Only coordinated effects were predicted by the then-prevailing view of oligopoly theory (Chamberlin 1929; 1933, chapter 3; Fellner 1949), which held that cooperation would tend to emerge spontaneously when the number of competitors was sufficiently small. The Cournot model was dismissed almost from the start as positing irrational behavior (see Fisher 1898, pp. 126–27). It was (mis)understood to assume that competitors myopically treated rival’s outputs as fixed, when in fact, each competitor’s output depended—even in the model—on the outputs of the others. Although Nash, non-cooperative equilibrium was well known by game theorists in the 1950s and 1960s, industrial organization economists did not
understand and embrace it until later, and only then did they appreciate the wisdom of the Cournot and Bertrand models (see Leonard 1994; Meyerson 1999).

As detailed by Werden (1997a), merger policy developed without any clear foundation in economic theory, but rather with a general abhorrence of industrial concentration. Columbia Steel (1948) introduced the term “relevant market” and was the first horizontal merger case to focus on market shares. Brown Shoe (1962, p. 335) held that “the proper definition of the market is a ‘necessary predicate’ to an examination of the competition that may be affected by the horizontal aspects of the merger.” And Philadelphia National Bank (1963, p. 363) established a presumption of illegality for “a merger which produces a firm controlling an undue percentage share of the relevant market, and results in a significant increase in the concentration of firms in the market.” These decisions remain significant, particularly because the U.S. Supreme Court has not had the occasion to address merger policy for three decades.

This paper first reviews the economic theory underlying the unilateral competitive effects of mergers, focusing on the Cournot model, commonly applied to homogeneous products; the Bertrand model, commonly applied to differentiated consumer products; and models of auctions and bargaining, commonly applied when a bidding process or negotiations are used to set prices. This paper then reviews the application of the theory through merger simulation.

Merger simulation calibrates a model of a one-shot, non-cooperative oligopoly game to match critical features of the industry, such as prices and outputs, then uses the calibrated model to compute the post-merger equilibrium that internalizes competition between the merging firms. With differentiated consumer products, to which merger simulation principally has been applied, calibration involves selecting values for the own- and cross-price elasticities of demand for relevant products. Typically used for this purpose are econometric estimates derived from high-frequency consumer purchase data. Over the past decade, merger simulation and econometric estimation of demand elasticities both have become common in the analysis of differentiated products mergers, and as Robert Willig (Merger Enforcement Workshop, Feb. 19, p. 124) declared, “the biggest change in the analytic framework used for merger enforcement has been the advent of simulation analysis.”
Theoretical Analysis of Unilateral Effects

A Formal Definition of Unilateral Effects

Consider an $n$-firm, simultaneous-move oligopoly game, in which competitors choose “actions” from the elements of the real line. Firm $i$’s action is $a_i$, and $a_{-i}$ is the $(n-1)$-tuple of actions taken by the other firms. The profits of firm $i$ are denoted $\Pi_i(a_i, a_{-i})$ to indicate their dependence on not just firm $i$’s own action, but also on the actions of its rivals. To ensure the existence and uniqueness of equilibrium, $\Pi_i(a_i, a_{-i})$ is assumed to be twice continuously differentiable and strictly concave.

The necessary and sufficient conditions for Nash equilibrium are

$$\frac{\partial \Pi_i(a_i, a_{-i})}{\partial a_i} = \frac{\partial \Pi_i'(a_i, a_{-i})}{\partial a_i} = 0.$$ 

In taking each of these partial derivatives, rivals’ actions are held constant, but that does not mean that firms actually treat rivals’ actions as fixed. For any actions by its rivals, firm $i$’s optimal action is given by its reaction function, or best-response function, which solves $\Pi_i'(a_i, a_{-i}) = 0$ for $a_i$ as a function of $a_{-i}$. Best-response functions may be implicit, although some profit functions permit an explicit solution

$$a_i = R_i(a_{-i}).$$

The Nash, non-cooperative equilibrium is the $n$-tuple of actions, such that each firm operates on its best-response function.

The merger of firms $i$ and $j$ produces a new firm choosing $a_i$ and $a_j$ to maximize $\Pi_i + \Pi_j$. For the merged firm, the necessary and sufficient conditions for Nash equilibrium are

$$\frac{\partial \Pi_i(a_i, a_{-i})}{\partial a_i} + \frac{\partial \Pi_j(a_i, a_{-i})}{\partial a_j} = 0,$$
$$\frac{\partial \Pi_i(a_i, a_{-i})}{\partial a_i} + \frac{\partial \Pi_j(a_i, a_{-i})}{\partial a_j} = 0.$$ 

The merger alters the optimal choice of $a_i$ and $a_j$ because the merged firm accounts for the effect of $a_i$ on $\Pi_j$ and the effect of $a_j$ on $\Pi_i$. Unless both effects are negligible, the merger affects the choice of both $a_i$ and $a_j$, and consequently, the actions of non-merging competitors as well, to the extent that the first derivatives of their profit functions are affected by changes in $a_i$ or $a_j$. The post-merger equilibrium fully reflects all firms’ responses to others’ responses and so forth.

The foregoing is typical of unilateral effects theories in merger cases, but what defines “unilateral effects” is simply that the strategies of non-merging firms are governed by the same, Nash-equilibrium, best-response functions before and after the
merger. Merger effects are unilateral even if the merged firm plays an oligopoly game different than that the merging firms had been playing (see, e.g., Daughety 1990 and Levin 1990), although that possibility is not considered below.

**Mergers in Cournot Industries**

**Mergers in the Basic Cournot Model.** In 1838 Antoine Augustin Cournot published the first systematic analysis of oligopoly, positing that the actions of competitors are their outputs. Cournot considered an industry with a homogeneous product, and it is to such an industry that the model generally is applied. In what follows, a “Cournot industry” is one in which competing producers of a homogeneous product simultaneously chose outputs in a one-shot game. Shapiro (1989, pp. 333–39) and Vives (1999, chapter 4) usefully analyze competition in a Cournot industry, and Szidarovszky and Yakowitz (1982) offer a technical treatment of the subject.

Firm $i$ in a Cournot industry has output $x_i$ and is completely characterized by its cost function $C_i(x_i)$. The industry has aggregate output $X$, and is characterized the set of incumbent competitors and the inverse demand $p = D(X)$. The profits of firm $i$ are

$$\Pi(x_i, X) = x_iD(X) - C_i(x_i).$$

Denoting the first derivatives of the demand and cost functions with primes, the necessary conditions for the Nash, non-cooperative equilibrium are

$$\frac{\partial \Pi(x_i, X)}{\partial x_i} = p + x_iD'(X) - C'_i(x_i) = 0.$$

If $\epsilon$ is the industry’s elasticity of demand, $m_i \equiv [p - C'_i(x_i)]/p$ is firm $i$’s price-cost margin, and $s_i \equiv x_i/X$ is firm $i$’s output share, these equilibrium conditions can be written

$$m_i = -s_i/\epsilon.$$

In equilibrium, therefore, the larger a firm’s output share, the larger is its margin and the lower its marginal cost.

If $m$ is the share-weighted industry average margin, and $H = \sum s_i^2$ is Herfindahl-Hirschman Index of output concentration, multiplying both sides of the equilibrium conditions by $s_i$, and summing over all firms in the industry, yields

$$m = -H/\epsilon.$$

If the pre- and post-merger demand elasticities are the same, and the merger does not affect the industry average marginal cost, it follows that
\[ \Delta p/p = -(H_{post} - H_{pre})/(\epsilon + H_{post}). \]

This result, however, is not useful for predicting the price effects of proposed mergers. If a merger is significantly anticompetitive, the unobservable, post-merger output shares that go into \( H_{post} \) are quite different than the observable, pre-merger output shares. Moreover, the assumption that the industry average marginal cost is constant cannot be maintained unless all firms in the industry have the same, constant marginal cost.

If the merged firm's output is \( x_m \) and its marginal cost is \( C'_m(\cdot) \), its equilibrium condition is

\[ p + x_m D'(X) - C'_m(x_m) = 0, \]

and the sum of the pre-merger equilibrium conditions for firms \( i \) and \( j \) is

\[ 2p + (x_i + x_j) D'(X) - C'_i(x_i) - C'_j(x_j) = 0. \]

Assuming the merger of those two firms induces neither the entry of a new competitor nor investments affecting the marginal costs of non-merging incumbents, the effect of the merger on price can be gleaned by subtracting the former condition from the latter. If \( x_i \) and \( x_j \) are the pre-merger outputs of the merging firms, the output of the merged firm is less than \( x_i + x_j \), and the merger increases price, unless

\[ C'_m(x_i + x_j) < C'_i(x_i) + C'_j(x_j) - p. \]

Substituting the conditions for pre-merger equilibrium margins into this inequality yields the conclusion that the merger increases price unless

\[ C'_m(x_i + x_j) < p[1 - (s_i + s_j)/\epsilon], \]

with the right-hand side of this inequality evaluated at the pre-merger equilibrium.

Farrell and Shapiro (1990a) derive this result and also demonstrate, as a general matter, that mergers in Cournot industries cause price to rise unless they generate sufficient off-setting synergies, in the sense that the merged firm can produce a given level of output at a lower total cost than the two separate merging firms could when optimally rationalizing production between them. Absent synergies, the merged firm chooses an output less than the sum of the outputs of the merging firms; the non-merging firms increase their output; but the net effect is a lower aggregate output and hence a higher price.

In a Cournot industry, Froeb and Werden (1998) show that

\[ -2s_i s_j/[\epsilon(s_i + s_j) + (s_i^2 + s_j^2)] \]
is the amount of the reduction in marginal cost, as a proportion of the share-weighted average of the merging firms’ marginal costs, required to prevent a merger from increasing price. As a proportion of the pre-merger price, the required reduction in marginal cost is

\[-2s_t s_j / \epsilon (s_i + s_j)\].

In the symmetric case, \(s_i = s_j = s\), so the former expression simplifies to \(-s / (\epsilon + s)\) and the latter to \(-s / \epsilon\), which equals the pre-merger margin of both merging firms. This final result provides a handy rule of thumb: To prevent a price increase following a merger in a Cournot industry, merger synergies must reduce the merged firm’s marginal cost, in absolute terms, by at least as much as the pre-merger price exceeds the merging firms’ marginal costs.

**The Significance of Cost Functions.** Just as firms in the Cournot model are characterized by their cost functions, the impact of a merger in the model is reflected in the difference between the cost function of the merged firm and those of the merging firms. If the merging firms have constant marginal cost and no capacity constraints, the merged firm is endowed with the lower of the two marginal costs. The effect of the merger is simply to destroy the higher-cost merging firm, and nothing of value is acquired when the lower-cost firm purchases the stock or assets of the higher-cost firm. While real-world corporate acquisitions on rare occasions may be designed to accomplish no more than destroying assets, that is certainly not the usual case. Hence, the Cournot model is apt to be of interest to merger policy only if marginal costs are increasing in the relevant range of output, or if there are significant capacity constraints. And as a description of real-world industries, the latter circumstance is apt to be more realistic than the former.

An interesting special case, analyzed by Werden (1991) and McAfee and Williams (1992), combines linear demand with quadratic costs of the form \(x_i^2 / 2k_i\), where \(k_i\) is a constant proportional to the capital stock or capacity of firm \(i\). The marginal cost of firm \(i\) is \(x_i / k_i\), and the merger of firms \(i\) and \(j\) combines their capital stock, yielding a marginal cost of \(x_m / (k_i + k_j)\). Defining, \(\kappa_i = k_i / (1 + k_i)\), \(\kappa_m = (k_i + k_j) / (1 + k_i + k_j)\), \(\kappa = \sum \kappa_i\), and \(\Delta \kappa = \kappa_m - \kappa_i - \kappa_j < 0\), it is easy to show that the proprotionate effect of the merger on the equilibrium price, \(\Delta p / p\), is given by \(-\Delta \kappa / (1 + \kappa_{mv} + \Delta \kappa)\). Thus, the larger \(\Delta \kappa\) in absolute value, the greater the effect of the merger on price. From this, Werden...
(1991) demonstrates that $\Delta p/p$ is increased by: (1) replacing either merging firm with a non-merging firm with a larger share of industry capital stock; (2) increasing the equality of the merging firm’s shares of industry capital, while holding their total capital constant; or (3) transferring capital between two non-merging firms in a manner that increases the inequality of their capital holdings.

**Profitability, Partial Acquisitions, and Entry.** The academic literature on mergers in Cournot industries has focused to a considerable extent on their profitability. Salant, Switzer, and Reynolds (1983) show that, in a symmetric Cournot industry, with linear demand, constant marginal costs, and no capacity constraints, mergers are unprofitable for the merging firms unless they involve at least eighty percent of the industry. But, as Perry and Porter (1985) note, those assumptions produce an unrealistic notion of a merger. Perry and Porter consider instead a model in which a firm’s marginal cost is increasing with a slope inversely proportional to a its capital stock, and they find greater scope for profitable mergers. In addition, Faulí-Oller (1997) and Hennessy (2000) show there is greater scope for profitable mergers in Cournot industries when demand is convex. Furthermore, the profitability of a real-world merger may derive from sources assumed away in these models, including cost reductions or other synergy gains in businesses of the merging firms other than those producing the anticompetitive effects.

Partial equity interests and joint ventures in Cournot industries are analyzed by Bresnahan and Salop (1986), Farrell and Shapiro (1990b), Flath (1992), Nye (1992), O’Brien and Salop (2000), and Reynolds and Snapp (1986). The main insight of this literature is that a purely financial interest in a competitor causes a firm to restrict its own output without directly affecting the behavior of the competitor in which the interest is held.

There also have been attempts to address the generally unrealistic assumption that competitors’ only actions are their outputs. Models have been analyzed in which competitors first make investment decisions determining their capacities, then choose prices. The equilibrium in such models depends on which consumers pay what price, when different firms charge different prices and the low-price firm cannot satisfy total market demand. Kreps and Scheinkman (1983) and Osborne and Pitchik (1986) show that the equilibrium is the same as that in the Cournot model if the available
low-price units of output are used to satisfy the demand of consumers willing to pay the most. Davidson and Deneckere (1986), however, show that first-come-first-served rationing produces an equilibrium more intensely competitive than the Cournot model.

Even if this last class of models yields the same equilibrium as the Cournot model, it does not follow that it makes the same predictions for the effects of mergers. If the pre-merger capacities are the Cournot outputs, it follows that non-merging firms are capacity constrained immediately following a merger. Considerable time may be required to adjust capacities to post-merger equilibrium levels, and until they adjust, the merged firm may find that a substantial price increase maximizes is short-run profits.

The literature has begun to explore the incentive for, and effects of, entry following merger in a Cournot industry. Werden and Froeb (1998) analyze a model in which entry after a merger necessarily results in a net price reduction, and in that model, a merger producing significant price increases is unlikely to induce entry. They also argue that the merging firms account for the effect of their merger on prospects for entry, so the decision to merge implies either the expectation of substantial cost reductions or the belief that significant entry would be unprofitable. Spector (2003) posits a model in which entry could leave the post-merger, post-entry price above the pre-merger level and shows that a merger is profitable, and thus occurs, only if any entry leaves price at a higher level. Gowrisankaran (1999) analyzes the very long-run outcome of a Cournot game with endogenous investment, entry, exit, and mergers. His numerical analysis suggests that mergers are likely to occur in Cournot industries (unless prevented by the legal system) and their price increasing effects are unlikely to be reversed by investment or entry.

**Mergers in Bertrand Industries**

**Mergers in the Basic Bertrand Model.** In an 1883 review of Cournot’s book, Joseph Louis François Bertrand posited that the actions of competitors are their prices, and competition in price has come to be known as Bertrand competition. Although Bertrand analyzed a homogeneous-product industry, Bertrand competition is considered most relevant for differentiated products. In what follows, a “Bertrand
industry” is one in which competitors simultaneously chose prices for competing brands of a differentiated product in a one-shot game. Vives (1999, chapter 6) usefully analyzes competition in a Bertrand industry. As Edgeworth (1925, pp. 116–21) pointed out, Bertrand competition may not produce an equilibrium in pure strategies, and conditions necessary to assure existence of equilibrium are assumed below.

Deneckere and Davidson (1985) provide a reasonably general proof that mergers in Bertrand industries raise prices and are profitable for the merging firms, although the mergers are even more profitable for the non-merging firms. The basic intuition for the price-raising effect of mergers can be gleaned from consideration of a Bertrand industry with single-brand firms. Brand i’s price is \( p_i \); the vector of prices for its competing brands is \( p_{-i} \); the demand for brand i is \( D_i(p_i, p_{-i}) \); and the cost of producing brand i is \( C_i(D_i(p_i, p_{-i})) \). Consequently, the profits for brand i are

\[
\Pi_i(p_i, p_{-i}) = p_iD_i(p_i, p_{-i}) - C_i(D_i(p_i, p_{-i})).
\]

The necessary conditions for the Nash, non-cooperative equilibrium are

\[
\frac{\partial \Pi_i(p_i, p_{-i})}{\partial p_i} = D_i(p_i, p_{-i}) + [p_i - C'(D_i(p_i, p_{-i}))] \frac{\partial D_i(p_i, p_{-i})}{\partial p_i} = 0.
\]

If \( \epsilon_{ij} \) is the elasticity of the demand for brand j with respect to the price of brand i, and \( m_i = [p_i - C'(D_i(p_i, p_{-i}))]/p_i \) is brand i’s price-cost margin, the necessary conditions can be written

\[
m_i = -1/\epsilon_{ii},
\]

which is the familiar inverse-elasticity rule or Lerner (1934) condition for equilibrium in monopoly.

If brands i and j are merged together, the post-merger necessary conditions for equilibrium are

\[
m_i = -1/\epsilon_{ii} + m_i d_{ij} p_i/p_j \]
\[
m_j = -1/\epsilon_{jj} + m_j d_{ij} p_j/p_i
\]

in which \( d_{ij} \) is the diversion ratio from brand i to brand j, that is, the ratio of the increase in quantity of brand j sold to the decrease in the quantity of brand i sold, when the price of brand i is increased slightly, formally,

\[
d_{ij} = -\epsilon_{ij} D_j/\epsilon_{ii} D_i.
\]

The last term in the equilibrium conditions is positive if brands i and j are substitutes. It follows that the merger raises the prices of both products unless it also reduces marginal costs or induces the entry of a new brand or investments altering consumer
perceptions about incumbent brands (termed “repositioning”). The post-merger equilibrium conditions for single-brand firms also provide useful intuition about what determines the magnitudes of the price increases. The price increases are greater, the greater are the diversion ratios and the pre-merger margins for the merging brands (and the less elastic their pre-merger demands). Shapiro (1996), who introduced the concept of the “diversion ratio,” usefully outlines the foregoing. The basic insights, however were already known (see, e.g., Werden 1982).

**The Significance of Demand Curvature.** In the symmetric case, with \( m_i = m_j = m \) and \( d_{ij} = d_{ji} = d \), there are simple expressions for the price effects of mergers of single-brand firms, provided the mergers do not affect costs and do not induce entry or repositioning. Shapiro (1996) shows that, with isoelastic demand,

\[
\frac{\Delta p}{p} = \frac{md}{(1-m-d)},
\]

and with linear demand,

\[
\frac{\Delta p}{p} = \frac{md}{2(1-d)}.
\]

These expressions illustrate the importance of the form of the demand curve for the price increases from differentiated products mergers. For all positive values for \( m \) and \( d \), isoelastic demand results in price increases more than twice those with linear demand. This demonstrates by example the importance of the higher-order or “curvature” properties of demand.

Froeb, Tschantz, and Werden (2004) systematically examine the impact of demand curvature on the price effects of mergers in Bertrand industries and on the extent to which marginal-cost reductions are passed through to consumers in the form of lower prices. The analysis is much like that for the monopoly case, which has the same inverse-elasticity equilibrium condition as a Bertrand industry made up of single-product firms. The only difference between the two cases is that, in a Bertrand industry, a reduction in the marginal cost of producing either of the merged firm’s brands affects the post-merger equilibrium prices of all competing brands, and these cross-price effects depend on idiosyncratic properties of the functional form of demand.

Letting \( \epsilon(p) \) be the elasticity of demand, the monopoly equilibrium condition can be written

\[
-\epsilon(p) = p/(p - c) \equiv 1/m.
\]
If $\eta(p) = \epsilon'(p)/\epsilon(p)$ is the elasticity of the elasticity of demand, and $\gamma$ is the derivative of the equilibrium price with respect to a constant marginal cost, total differentiation of the equilibrium condition yields:

$$\gamma = 1/[1 + (\eta(p) - 1)m] = \epsilon(p)/[1 + \epsilon(p) - \eta(p)].$$

This result is derived by Bulow and Pfleiderer (1983) (although printed with plus sign where the minus sign should be). It follows that:

- $\gamma < 1$ if $\eta(p) > 1$, as with linear demand, for which $\gamma = \frac{1}{2}$;
- $\gamma = 1$ if $\eta(p) = 1$, as with $D(p) = a \exp(-bp)$ when $a, b > 0$; and
- $\gamma > 1$ if $\eta(p) < 1$, as with isoelastic demand.

It is also self-evident that the monopoly-equilibrium margin is higher, the lower is the elasticity of the elasticity of demand.

While both the price and pass-through effects of mergers in Bertrand industries depend on the curvature of demand, not dependent on demand curvature is the magnitude of marginal-cost reductions that exactly restore pre-merger prices, assuming the merger induces neither the entry of a new brand nor repositioning of existing brands. Because all prices remain the same, so too do all demand elasticities, which makes these compensating marginal cost reductions (CMCRs) robust to the form of demand. Werden (1998) derives a general expression for the CMCRs. Defining $m$ and $d$ as before, the CMCRs for both merging brands in the symmetric case are

$$md/(1-m)(1-d),$$

when expressed as a proportion of pre-merger marginal cost. Expressed instead as a proportion of pre-merger price, the CMCRs are

$$md/(1-d).$$

If $d = \frac{1}{2}$, making the merging brands exceptionally close substitutes, this implies the same rule of thumb as in a Cournot industry: To prevent post-merger price increases, the marginal costs of both the merging brands must fall in absolute terms by at least as much as the pre-merger prices exceed the pre-merger marginal costs. Typically, merging brands are far less close substitutes, so smaller cost reductions prevent price increases.

**Bertrand Mergers with Logit Demand.** Werden and Froeb (1994) analyze mergers among single-brand competitors in the context of logit demand, which generally is
motivated by a random utility model of consumer choice. As formalized by Manski
(1977), each consumer in this class of models makes a single choice from an
exhaustive set of alternatives, \( A \), consisting of some particular alternatives plus “none
of the above.” In this formulation, every alternative in \( A \) is necessarily a substitute for
all the others. Consumers maximize utility, and the utility associated with each
alternative is the sum of a “systematic” or “representative” component, \( V_i \), common
to all consumers, and a component specific to the individual consumer, which is
treated as random to the outside observer. If and only if the random component of
utility has the Type 1 extreme value distribution, McFadden (1974, pp. 111–12) and
Anderson, de Palma, and Thisse (1992, pp. 39–40) have shown that result is the logit
model, in which case, the probability of choosing brand \( i \), over the entire population
of consumers, is

\[
\pi_i = \frac{\exp(V_i)}{\sum_{k \in A} \exp(V_k)}.
\]

The simplest logit model specifies the systematic component of utility as

\[
V_i = \alpha_i - \beta p_i,
\]

in which \( \alpha_i \) is a constant that indicates, roughly, brand \( i \)’s average preference and \( \beta \)
is a constant that determines the degree of substitutability among alternatives. The
own-price elasticity of demand for brand \( i \) is

\[-\beta p_i (1-\pi_i),\]

and the cross-price elasticity of the demand for brand \( i \) with respect to the price of
brand \( j \) is

\[\beta p_i \pi_j.\]

With a very large \( \beta \), competing brands are nearly perfect substitutes, so competition
from non-merging brands prevents a merger from increasing prices. And with a very
small \( \beta \), there is essentially no competition to lose from merging two brands.

With single-brand firms, logit demand, and constant marginal costs, \( c_i \), Werden
and Froeb (1994) show that necessary conditions for Bertrand-Nash equilibrium are

\[p_i - c_i = 1/\beta (1-\pi_i).\]

Before the merger, the mark-up for brand \( i \), \( p_i - c_i \), is higher the larger is its choice
probability. The necessary conditions for equilibrium for the firm formed by merging
brands \( i \) and \( j \) are

\[p_i - c_i = p_j - c_j = 1/\beta (1-\pi_i-\pi_j).\]
Because both of the merged firm’s brands have the same mark-up, it follows that the merger causes a larger increase (in absolute terms) in the price of the merging brand with the smaller pre-merger choice probability. The primary reason for this relates to pattern of switching between the two brands in response to a price increase: For any given loss in sales from a price increase for the merging brands, a larger portion is recaptured by the brand with the larger choice probability. A second reason is that the brand with the larger choice probability has the larger mark-up in the pre-merger equilibrium, making any given sales recapture more profitable. The same factors apply in general, although without logit demand, they may not be controlling.

Werden and Froeb (1994) also show that the slope of the best-response function for non-merging brand \( k \), to an increase in price of merging product \( i \), is given by 
\[
\frac{\pi_i}{\pi_k} / (1 - \pi_k).
\]
This expression is positive and increasing in both choice probabilities. Consequently, the prices of non-merging brands increase in response to increases in the prices of merging brands, with greater increases for brands with larger pre-merger choice probabilities. For non-merging brands with choice probabilities less than those of both merging brands, the slope of the best-response function is less than one-sixth, so the prices of non-merging brands are apt to increase much less than those of the merging brands, and that typically is the case with other demand assumptions.

**Independence of Irrelevant Alternatives and Closeness of Substitutes.** The logit model exhibits Independence of Irrelevant Alternatives (IIA), i.e., for any alternatives \( i \) and \( j \) and any subset, \( S \), of the choice set \( A \),
\[
\frac{\text{Prob}(i|S)}{\text{Prob}(j|S)} = \frac{\text{Prob}(i|A)}{\text{Prob}(j|A)}.
\]
The IIA property was introduced by Luce (1959, pp. 5–6, 12–15), who termed it the “choice axiom” and found it consistent with some experimental evidence. Debreu (1960) immediately noted that the IIA property cannot hold in some choice problems, and much economic literature has considered the IIA property unreasonably restrictive.

The IIA property, however, can be made far less restrictive by formulating the choice problem so that there is only a limited range of choices over which the IIA property is assumed to apply. One way to do this is to model the choice problem hierarchically, as in the nested version of the logit model. The IIA property then is
assumed to apply at each stage of the choice problem. A very simple way to limit the range of choices over which the IIA property is assumed to apply is to model only a portion of the choice problem. For example, rather than modeling choice among all automobiles, it may suffice to model choice among just economy cars.

In practical terms, the IIA property implies substitution proportionate to relative choice probabilities: If alternatives $i$ and $j$ have choice probabilities .3 and .1, the IIA property implies that an increase in the price of any other alternative in the choice set necessarily induces three times as much substitution to $i$ as to $j$. Because an increase in the price of one alternative induces equi-proportionate increases in the consumption of all other alternatives in the choice set, there are equal cross-price elasticities of demand for every alternative in the choice set with respect to the price of a given alternative.

The IIA property provides the most useful definition of what it means for all alternatives to be equally close substitutes for each other. There are, however, other possible definitions. For example, one might say all alternatives are equally close substitutes if an increase in the price of one induces equal absolute increases in the consumption (or the value of the consumption) of all other alternatives in the choice set.

A common misconception among lawyers and reflected by the court’s analysis in Oracle (2004, pp. 1117, 1166–68, 1172–73) is that mergers have significant unilateral anticompetitive effects in Bertrand industries only when competition is highly localized, as in a spatial model, the merger involves adjacent brands, and the merging brands are widely separated from non-merging brands. As correctly stated by the Horizontal Merger Guidelines (1992, § 2.21): “Substantial unilateral price elevation in a market for differentiated products requires that there be a significant share of sales in the market accounted for by consumers who regard the products of the merging firms as their first and second choices . . . .” No matter what metric may be used to define closeness of substitutes, this condition can be satisfied and a substantial anticompetitive effect can occur, even if the merging brands are not next-closest substitutes as a general matter, rather than in just the view of particular customers.

**Entry, Repositioning, and Other Dimensions of Competition.** Werden and Froeb (1998) explore the possibility of entry following merger in randomly generated
Bertrand industries with logit demand. Their results suggest that mergers are unlikely to induce entry unless entry was nearly profitable pre-merger. Cabral (2003) considers the possibility of entry following mergers in a spatially differentiated Bertrand industry, and finds that merger to monopoly is reasonably likely to induce entry, leading to prices lower than they were pre-merger.

Thus far it has been assumed that price is the only dimension of competition, but price is only one of several dimensions of competition in consumer products industries—a fact stressed in the field of marketing. Little work has been done on the effects of mergers among competitors that choose both price and another strategic variable, such as product positioning or the level of promotion. Analyzing a merger solely in terms of price competition seems likely to be a reasonable simplification in many cases, but little theoretical or empirical analysis currently supports that view.

Gandhi et al. (2004) analyze mergers with competition in both price and location. Location in their model is along the Hotelling line, and consumer choice is determined both by distance and random component capturing idiosyncratic tastes. They find that all competitors raise price following a merger, just as in a conventional Bertrand industry, but competitors also change locations, which feeds back into pricing. The merging firms move away from each other, to avoid cannibalizing each others’ sales. The non-merging firms alter their locations in response, occupying space the merging firms vacate. Repositioning increases variety in a way that benefits consumers and thereby offsets the loss in consumer welfare from price increases. The nature of the product repositioning, however, is not as contemplated in the Horizontal Merger Guidelines. Non-merging brands are not repositioned closer to the merging brands in a manner that mitigates the loss of competition from the merger.

Gandhi et al. assume that changing location is costless, which is highly unrealistic in many industries. Altering product characteristics can be both expensive and risky, and it can take sufficiently long that ignoring that possibility altogether yields a good prediction of a merger’s short-term effects. However, Berry and Waldfogel (2001) find that mergers among radio stations resulted in increased format variety within a short period of time. The increase in variety likely benefitted listeners, but it may have harmed advertisers by making radio stations more distant substitutes and thereby leading to an increase in advertising rates.
Froeb, Tenn, and Tschantz (2004) analyze mergers with competition in both price and “advertising,” which can be thought of as any form of non-price promotion affecting demand. They show that erroneously modeling competition as being just in price can yield an under- or over-prediction of a merger’s price effects, depending on how optimal advertising expenditures vary with price. For example, if a higher price caused a lower level of advertising to be preferred, the predicted price effect of a merger, ignoring competition in advertising, would be too low because the post-merger price increases would induce a reduction in advertising, which would lead to still higher prices. How optimal advertising expenditures vary with price, of course, is an empirical question, as is whether they interact strongly enough to matter.

Mergers in Auction Models

The Cournot and Bertrand models characterize equilibrium outcomes of a competitive process without indicating how the process works and produces those outcomes. Auctions models are essentially Bertrand models augmented by rigid specifications for how bidders interact under rules dictated by the auctioneer. William Vickery (1961, 1962) initially formalized the analysis of competition in a bidding setting, with significant elaboration provided in particular by Milgrom and Weber (1982). The vast auction literature is usefully summarized by Klemperer (1999, 2004).

Auction models differ depending on the detailed specifications of the bidding process, and to simplify the discussion, this paper primarily considers the ascending-bid, oral auction, or English auction. Competition in an auction can be among buyers or sellers, and this paper focuses initially on auctions in which bidders compete to purchase an item. Auction models are broadly divided into “private values” and “common values” models, and this paper considers only the former. Bidders in private values purchasing auctions are distinguished by the valuations they place on the item auctioned.

Mergers in Private Values English Auctions. In a private values English auction, each bidder’s optimal non-cooperative strategy is to bid up to the valuation placed on the item. Consider bidders with valuations of 1, 2, 3, and 4. As the level of bidding
ascends, the first bidder drops out at 1, the second at 2, and the third at 3. When the second-last bidder drops out at 3, the auction is over. What determines the winning bid is not how much the winner values the item; rather, it is how much the losers—as a group—value it. The number of bidders per se has no effect on the winning bid, but adding or subtracting bidders matters if that changes the second-highest valuation.

The usual way to model a bidder merger is to assume they form a coalition eliminating competition between them. It is readily apparent that a coalition affects the winning bid if, and only if, it contains the bidders with the first- and second-highest valuations. In the simple example, formation of a coalition of the bidders with valuations 3 and 4 would result in a winning bid of 2 instead of 3, but any coalition not including both of those bidders would not affect the winning bid. Mailith and Zemsky (1991) show that a bidding coalition in an English auction cannot be unprofitable.

The foregoing considers a single auction, but an industry is likely to have many separate auctions per year, perhaps thousands, with bidder valuations varying significantly across auctions. A bidder merger in such an industry would affect those auctions, and only those auctions, in which the merging firms have the first- and second-highest valuations. The average or expected effect of the merger is the frequency with which the merging bidders have the two highest valuations multiplied by the difference between the second- and third-highest valuations when they do.

An industry in which competition occurs through private values English auctions can be characterized by the distributions of bidders’ valuations. The effects of mergers in English auctions are much like the effects of mergers in Bertrand industries, and the joint distribution of bidders’ valuations plays much the same role as the demand function in the differentiated products context. In both cases, functional form matters.

**Mergers with Power-Related Distributions.** To analyze mergers in an auction setting, bidder asymmetry must be modeled in a tractable manner. One useful approach is to imagine each bidder taking multiple draws from a common distribution, $F(v)$, with that bidder’s valuation being the maximum from its draws. The greater number of draws a bidder takes, the greater is its expected valuation and
the greater is its probability of winning. Froeb and Tschantz (2002) and Waehrer and Perry (2003) analyze the effects of mergers in auctions modeled in this way, making use of the convenient properties of “power-related” distributions.

Cumulative valuation distribution functions \( F_1(v) \) and \( F_2(v) \) are power related if there exists a positive number \( r \) such that \( F_1(v) = [F_2(v)]^{r} = F_2^{r}(v) \), for all \( v \). The maximum value from multiple draws is in the same family of power-related valuation distributions as the distribution from which the draws are taken. Distributions that have been used in the literature because of their tractability are the uniform distribution and the Type 1 extreme value distribution, which gives rise to the logit model.

If \( F(v) \) is the parent cumulative valuation distribution function, the private valuation of bidder \( i, v_i \), has the distribution \( F(v) \) raised to the power \( t_i \), which reflects the size or strength of bidder \( i \). If bidders’ valuations are mutually independent and \( t = \sum t_i \), the distribution of the maximum valuation across all bidders is \( F'(v) = F_{\text{max}}(v_{\text{max}}) \). Waehrer and Perry (2003) show that the probability that bidder \( i \) draws the maximum value, and wins the auction, is \( \pi_i = t_i/t \). They motivate the \( t_i \)'s as bidder “capacities” and interpret this result as demonstrating that capacity shares are the same as winning shares. It follows immediately from their result that the IIA property holds: \( \pi_i/\pi_j \) is invariant to \( t \) and hence to presence or absence of other bidders. Consequently, \( \pi_i/\pi_j/(1-\pi_i) \) is the probability of bidder \( i \) making the highest bid when \( j \) makes the second-highest bid.

The maximum valuation of the losing bidders when bidder \( i \)'s wins, \( v_{-i} = \max(v_{i}, \ldots, v_{i-1}, v_{i+1}, \ldots, v_n) \), has the distribution \( F_{-i}(v_{-i}) \). Froeb, Tschantz, and Crooke (2001) show that the distribution of the winning bid when bidder \( i \) wins, \( B_i \), is
\[
F(B_i) = F(v_{-i} | v_i > v_{-i}) = F_{\text{max}}(v_{\text{max}}) - [F_{\text{max}}(v_{\text{max}}) - F_{-i}(v_{-i})]/\pi_i.
\]

The relationship between the probability of winning and the expected winning bid follows immediately. If \( \mu(r) \) is the expected value of the maximum valuation from the distribution \( F'(v) \), and \( t_{-i} = t - t_i \)
\[
E[B_i] = E[v_{-i} | v_i > v_{-i}] = \mu(t) - [\mu(t) - \mu(t_{-i})]/\pi_i.
\]

While not obvious from this moment condition, the expected winning bid for any particular bidder is lower the higher that bidder’s probability of winning. It is much easier to see that more successful bidders earn higher expected profits. Bidder \( i \)'s
expected profit is its winning probability times the expected difference between its valuation and its winning bid:

\[ \pi_i E[v_{\text{max}} - B_j] = \mu(t) - \mu(t) = \mu(t) - \mu(t(1 - \pi_j)) \equiv h(\pi_i). \]

This result is analogous to results in the Cournot and Bertrand models relating equilibrium margins to equilibrium shares and own-elasticities of demand. The total expected profit to all bidders is the sum of the \( h(\pi_i) \).

Bidders \( i \) and \( j \) have probabilities of winning \( \pi_i = t_i/t \) and \( \pi_j = t_j/t \), and if they merge, the merged firm has the probability of winning \((t_i + t_j)/t = \pi_i + \pi_j\). Thus, the merger causes the combined expected profits of these bidders to increase by

\[ h(\pi_i + \pi_j) - h(\pi_i) - h(\pi_j). \]

English auctions are efficient in the sense that the bidder with the highest valuation always wins the auction, so the short-term effect of a merger is merely to transfer rents, and the expected increase in the merging bidders’ profits equals the expected effect of the merger on winning bids. As Brannman and Froeb (2000) illustrate graphically, the expected price effect of a merger is a function of pre-merger market shares, and that function is determined by the curvature of the \( h(\cdot) \) function, which is determined by the underlying distribution \( F(v) \).

The analysis here is based on models without a reserve price. Waehrer and Perry (2003) show that the use of a reserve price can cause the expected winning bid to increase, rather than decrease, following a merger in a private values English auction, because of an effect on the optimal reserve price.

**Two Special Cases.** One interesting special case uses as the parent distribution the Type 1 extreme value distribution used above to generate the logit model. Froeb, Tschantz, and Crooke (2001) construct a logit auction model by combining the distribution \( F(v) = \exp[-\exp(-\lambda v)] \) with \( t_i = \exp(\lambda \zeta_i) \), so that

\[ F_i(v) = \exp[-\exp(-\lambda(v - \zeta_i))]. \]

The winning probabilities take the familiar logit form:

\[ \pi_i = \exp(\lambda \zeta_i) / \sum \exp(\lambda \zeta_i). \]

The maximum of independent draws from this extreme value distribution also has an extreme value distribution, and it has the same variance as distribution from which the draws are taken. Defining \( \sigma^2 = 6 \text{Var}(v) / \pi^2 \), Froeb, Tschantz, and Crooke (2001) show that
\( h(\pi_i) = -\sigma \log(1-\pi_i). \)

This function is increasing in the variance of the underlying valuation distribution because a higher variance causes a greater expected spread between the two-highest valuations. The effect of a merger of bidders \( i \) and \( j \) on expected profits is

\[-\sigma [\log(1-\pi_i - \pi_j) - \log(1-\pi_i) - \log(1-\pi_j)].\]

A higher variance of the underlying valuation distribution causes a greater effect from the merger because it causes a greater expected spread between the second- and third-highest valuations. The specific effect of the merger is determined by curvature properties, just as in a Bertrand industry.

The uniform distribution:

\[ F(v) = (v-a)/(b-a), \quad a \leq v \leq b, \]

is the parent for another power-related family of distributions. Froeb, Tschantz, and Crooke (2001) show that the valuation distribution for a bidder taking \( t_i \) draws has the mean \((a + bt_i)/(t_i + 1)\), and the variance \((b-a)^2 t_i / (t_i + 1)^2(t_i + 2)\), and for that bidder,

\[ h(\pi_i) = \pi_i(b-a)t_i / [(t_i + 1)(t_i(1-\pi_i) + 1)]. \]

The dependence of the profit function on the variance is seen in the term \((b-a)\), the range of the support for \( v \). This function can be used to calculate the effect of a merger, which again depends on its curvature properties.

**Mergers in Procurement and Sealed-Bid Auctions.** The foregoing analysis changes little when adapted to procurement auctions, in which bidders typically are modeled as drawing the cost of serving a particular customer from some distribution. The results of the two previous sections with power-related distributions are easily adapted, and Waehrer and Perry (2003) analyze the effects of mergers with power-related cost distributions. Cumulative cost distribution functions \( F_1(c) \) and \( F_2(c) \) are power related if there exists a positive number \( r \) such that \( F_1(c) = 1 - [1 - F_2(c)]^r \), for all \( c \). \( F(B_i) \) and \( E[B_i] \) are exactly the same as before, except that the valuation distributions are replaced by cost distributions and the maximum valuation is replaced by the minimum cost. Bidder \( i \)'s expected profit is

\[ \pi_i E[B_i - c_{min}] = \mu(t_i) - \mu(t) = \mu(t(1-\pi_i)) - \mu(t) \equiv h(\pi_i). \]

The \( h(\cdot) \) functions for the Type 1 extreme value and uniform distributions are exactly the same as before although the means of the distributions are not. For example, if \( F(c) \) is uniform, the cost distribution for a bidder taking \( t_i \) draws has the mean...
\( \frac{at + b}{t + 1} \).

Procurement auctions also can be modeled as competitions among differentiated products: Buyers with heterogeneous tastes or needs perceive differences in the products of competing bidders. Bidders draw valuations, much as in purchasing auctions, except that the valuations relate to a particular customer’s utility from a particular bidder’s product. With such auctions, the results above carry over with appropriate switching of signs.

In a sealed-bid, first-price procurement auction, the winner pays the amount actually bid rather than the amount of the second-highest bid. This alters bidders’ profit calculus and consequently, their optimal bids. In order for the expected pay-off from participating in the auction to be positive, bids must be less than valuations. Although bidders find it optimal to bid differently in oral auctions than in sealed-bid auctions, the Revenue Equivalence Theorem, originally proved by Vickery (1961) and later generalized, states that the expected winning bid is the same for the two types of auctions. Proofs of the theorem, however, assume symmetric bidders, and Maskin and Riley (2000) show that the Theorem does not hold with asymmetric bidders. Rather, either auction may result in the higher expected winning bid, depending how the valuation distributions of bidders relate to each other.

Because bidding functions depend on the specific valuation distributions assumed, there are few general results for asymmetric sealed-bid auctions. Marshall et al. (1994) and Dalkir, Logan, and Masson (2000) compute sealed-bid equilibria using power-related uniform distributions. Tschantz, Crooke, and Froeb (2000) do so for the logit auction model. Conditioning on the merging firms’ pre-merger shares, they find that the price effects of mergers in a sealed bid auction are almost perfectly predicted by taking 85% of the price effect predicted using the oral auction model.

**Mergers in Auctions vs. Mergers in Bertrand Industries.** There is much in common between the effects of mergers in industries characterized by the private values English auction model and their effects in industries characterized by the Bertrand model, but there are differences as well. All customers face the same prices in a Bertrand model, but each faces its own set of prices in an auction model. Consequently, the unilateral exercise of market power following a merger in a private values English auction industry does not affect the identity of the winner bidder in
any auction. In contrast, the unilateral exercise of market power following a merger in a Bertrand industry causes some customers to switch their purchases. In the short run, there is a reduction in total surplus from a merger in a Bertrand industry but not in an industry employing English auctions. The effect of synergies also differs. In a Bertrand industry, reduced marginal costs for the merging brands are passed through to some extent in the form of lower prices on those brands. In an auction model, a marginal-cost reduction has no affect on prices when the merged firm wins the auction, but rather affects price when the merged firm has the second-highest valuation and loses the auction.

In some auction models (e.g., Brannman and Froeb 2000), it has been observed that substantial price effects from mergers leaving at least two bidders require an implausibly high variance in the underlying valuation or cost distribution. But this does not suggest that auctions are inherently more competitive than Bertrand industries. Rather, the substantial degree of differentiation in many industries to which the Bertrand model is applied is not matched by many of the industries to which auction models are applied.

**Mergers in Bargaining Models**

John F. Nash, Jr. (1950, 1953) originally developed the theory of bargaining, and Osborne and Rubinstein (1990) usefully present both Nash’s contributions and subsequent developments. From first principles, Nash’s axiomatic bargaining theory identifies a reasonable solution to the bargaining game. In sharp contrast to auction models, this abstracts from the process of bargaining completely. Strategic bargaining theory, however, resembles auction theory in its attention to the details of how the game is played.

The simplest strategic bargaining game involves splitting a pie. Suppose players take turns offering splits and the game ends either when an offer is accepted, or after the third round if no offer is accepted, in which case neither player gets anything. The Nash, non-cooperative equilibrium is identified by backward induction. Whichever player makes the offer in round three offers the other player nothing, which is what that player would get by rejecting the offer. In round two, the player that will make the offer in round three rejects any offer of less than the whole pie, because that
guarantees getting the whole pie in round three. In round one, the optimal offer is again to give the other player nothing, because the same player makes the offer in round one as in round three, and the other player gets nothing later if the offer in round one is rejected.

The equilibrium to this strategic bargaining game is drastically different if order of play or number of rounds is changed, yet the real world normally has no fixed number of rounds nor a designated first mover. This contrasts with an auction, in which the auctioneer can dictate the rules and commit to a take-it-or-leave-it offer. In order to avoid sensitivity to arbitrary conditions, Nash (1953) posited axioms for reasonable solutions to the bargaining game, including that the outcome could not depend on which player moves first, and the IIA property. If \( z \) is the outcome of the bargaining game, \( V_i(z) \) is the value players place on outcome \( z \), and \( d_i \) is “disagreement value,” or pay-off if no bargain is reached, Nash showed that any solution satisfying these axioms maximizes the value of

\[
[V_i(z) - d_i] [V_j(z) - d_j],
\]

which is the product of the incremental surpluses the players derive from reaching a bargain. In the example of splitting a pie, the disagreement values are zero, and the problem is to maximize \( z(1-z) \). The solution is that each gets half.

Nash hypothesized that his axiomatic solution was also the equilibrium of some larger strategic game, and Rubinstein (1982) took a major step toward confirming Nash’s hypothesis by identifying the subgame-perfect equilibrium in the infinite-round, alternating-offer game to split a pie. Assuming the value of the pie at the end of each period is \( \delta < 1 \) times its value at the beginning, Rubinstein proves that equilibrium fraction of the pie going to player making the first offer is

\[
(1 - \delta)/(1 - \delta^2).
\]

For discount factors close to 1, this is close to Nash’s axiomatic solution of \( \frac{1}{2} \). The player making the first offer can bargain for slightly more than half of the pie because the value of pie declines between that first offer and any counter by other player. Binmore, Rubinstein, and Wolinsky (1986) show that the unique subgame-perfect equilibrium of the alternating-offer game converges to Nash’s axiomatic solution as the time period between offers, and hence the decline in the pie’s value, approaches zero.
Nash’s axiomatic solution provides intuition as to how a merger may affect the outcome of bargaining, and one possibility is no effect at all. Suppose players $A_1, \ldots, A_n$ bargain with their counterparts $B_1, \ldots, B_n$ over the splitting of $n$ pies, and assume that two or more of the $A$ players form a coalition. This coalition does not affect the splitting of any of the pies because it has no effect on the incremental surplus to either the $A$ players or the $B$ players from striking a bargain. But some mergers do affect the incremental surplus of either the merged firm or the party with which it is bargaining.

Vistnes (2000) explains that the merger of two hospitals may allow them to achieve a better bargain if the hospitals are alternatives for the networks of managed care plans. The merger increases the plans’ incremental surplus from striking a bargain because it means that the other merging hospital also will not be in the network if no bargain is struck. Raskovich (2003) shows that the roughly the reverse also may occur if merging firms bargain with a supplier with substantial fixed, but not yet sunk, costs. He posits that the merged firm becomes “pivotal” to the supplier, in the sense that the supplier can cover its fixed costs only by striking a bargain with the merged firm. The merger, therefore, reduces the supplier’s incremental surplus from striking a bargain with the merging firms and allows the supplier to achieve a better bargain.

A merger that reduces the marginal cost of supplying a particular customer increases the incremental surplus to the merged firm from striking a bargain with that customer, causing marginal-cost reductions to be partially passed through. The rate of pass-through in Nash’s axiomatic solution is determined by the curvature of value functions. In a constant-sum game, such as splitting a pie, Nash’s axiomatic bargaining solution yields a pass-through rate of 50%, just as in a Bertrand industry with linear demand. Moreover, savings of fixed, but not sunk, costs may be passed through in Nash’s solution, although not in a Bertrand or Cournot industry. If some customers are large enough that there are recurring fixed costs associated with their particular accounts, merger-related reductions in those fixed costs are shared with the customers.
Merger Simulation

The Rationale for, and Mechanics of, Merger Simulation

The Rationale of Merger Simulation. Merger simulation has been used primarily with differentiated consumer products, and the rationale for its use in that context is illustrated by the Kraft (1995) case, in which the court rejected a challenge to Kraft’s consummated acquisition of Nabisco. To assess the unilateral effects of the combination of Post Grape-Nuts, owned by Kraft, with Nabisco Shredded Wheat, the economic expert testifying for the merging firms estimated the relevant elasticities of demand. He opined that the cross-price elasticities of demand between the two brands were too low to produce significant anticompetitive effects (see Rubinfeld 2000), and the court agreed.

A subsequent analysis by Nevo (2000b), based on his own elasticity estimates, predicted price increases of 3.1% and 1.5% for Shredded Wheat and Grape Nuts, absent any marginal-cost reductions. It is unclear how these predictions compare to what either the expert or the court gleaned from the raw cross-price elasticity estimates; both may well have believed that much smaller price increases were implied. The key point, however, is that the court did not have the benefit of any systematic analysis of the implications of the elasticity estimates. Though it may have been correct, the court’s decision was uninformed. Merger simulation could have usefully substituted objective and verifiable calculations for subjective and unverifiable intuition.

Merger simulation is also particularly useful with differentiated consumer products because that is the context in which the focus on market shares and concentration is most problematic in the traditional legal analysis of mergers. The competitive effect of merging two differentiated product depends largely on the cross-price elasticities of demand between those products, which are only very roughly suggested by market shares. Werden and Froeb (1996) illustrate the point by simulating mergers in randomly generated industries with logit demand. Logit demand makes shares the best predictors they can be, because it causes substitution away from any brand to be distributed among competing brands in proportion to their relative shares. Nevertheless, Werden and Froeb find a huge variance in the price effects of differentiated products mergers for a given set of market shares,
depending on the values of the parameters of the logit demand function.

Werden and Rozanski (1994) explain why market delineation is likely to obscure more than it illuminates when highly differentiated consumer products are involved. Consumers typically have differing and complex preferences, and available alternatives appear over a broad and fairly continuous range of prices and attributes. Under these conditions, the merging firms often argue that no meaningful boundaries can be drawn within a price and quality continuum, as the court found in *Brown Shoe* (1963, p. 326), although shares of a very broadly delineated market may mask an intense competitive interaction between the merging brands. And if the merging brands are particularly close substitutes, the plaintiff generally argues for a very narrow market, although shares in such a market ignore the potentially significant competitive impact of brands outside the delineated market.

A major advantage of merger simulation with differentiated consumer products is that it eliminates the need for market delineation. It is necessary to designate which brands are included in a merger simulation, but the competitive significance of a brand is accounted for, even if it is excluded. In the context of merger simulation, the word “shares” refers to included-brand shares. Relative shares matter, but they are invariant to which brands are included.

Only included brands interact strategically, so only their prices may change as a result of the merger. The prices of excluded brands are held constant, and competition from them is incorporated through the own-price elasticities of demand of the included brands. As indicated by Werden and Froeb (1994), the predicted effects of mergers generally are insensitive to which non-merging brands are included. The reason is that the prices of non-merging brands generally change far less, often an order of magnitude less, than those of the merging brands. It is best, however, to include a non-merging brand if a large fraction of purchasers of a merging brands would substitute to it in the event of a price increase.

Merger simulation also permits an explicit trade-off between the anticompetitive effects of internalizing competition between the merging products and the pro-competitive effects of cost reductions resulting from the merger. In a traditional analysis focusing on market shares, there is no explicit way to trade-off cost reductions, no matter what sort of trade-off is deemed appropriate. With merger
simulation, it is simple to predict the price effects of a merger after accounting for any marginal-cost reductions, and under certain demand assumptions, it is easy to compute the net effect of a merger on standard measures of economic welfare. Similarly, it is straightforward to predict the net effect of both a merger and a curative divestiture, as done by Jayaratne and Shapiro (2000).

Like any formal modeling, merger simulation forces assumptions to be made explicit. That, in turn, adds focus to the analysis by identifying what really matters, why it matters, and how much it matters. Performing simulation helps guide a merger investigation by indicating both the kinds of evidence is useful to gather and how to interpret what has been gathered. At the same time, the evidence gathered indicates what modeling assumptions are appropriate. While merger simulation applies abstract theoretical models, its proper use assures that the specific facts of each case play a central role in merger analysis. Merger simulation clarifies the implications of established facts for the likely unilateral effects of proposed mergers, by combining those facts with reasonable assumptions about what is not known, and evaluating their significance in a precise, objective manner.

Notwithstanding all of the foregoing advantages of merger simulation, there are serious limitations. The predictions of merger simulation are at best reasonable, but rough, estimates of the likely effects of mergers. Price-increase predictions always are subject to modeling error, stemming from assumptions that are never exactly right and may be terribly wrong, as well as from sampling error in the statistical estimation of model parameters. These two sources of error imply, for example, that price increase predictions close to zero cannot meaningfully be distinguished from zero.

At the current state of the art, merger simulation also predicts only the immediate price and output effects of mergers. Issues relating to the longer-term evolution of the industry, such as entry, product repositioning, and other changes in marketing strategy are assumed away in merger simulation and hence must be separately considered. The limitations of merger simulation must be assessed within the factual context of any particular case.

The Mechanics of Cournot Merger Simulation. The Cournot model is rarely used to simulate mergers, but its simplicity of makes it useful for introducing the basic ideas, mechanics, and capabilities of merger simulation. Assuming that a proposed
merger would occur in Cournot industry, simulating its effects proceeds in three steps: (1) specification of functional forms for demand and cost, (2) calibration of the parameters of these functions to make them fit the pre-merger equilibrium, and (3) computation of the post-merger equilibrium.

Because a Cournot industry is characterized by the demand for the industry and the cost functions of the firms, the first step in simulating a Cournot merger is specifying functional forms for both. With today’s desktop computing power, computation places no constraints on functional form, but calibration may. The less about demand and cost that can be observed or inferred, the greater the structure that must be imposed through the assumption of simple functional forms. One of the virtues of merger simulation is that strong assumptions permit calibration from information likely to be amassed in the typical merger investigation. One of the main limitations of merger simulation is that its predictions can be highly sensitive to these assumptions.

A merger investigation can be expected to yield basic descriptive information about an industry, including an average price, an aggregate annual output, and the output shares of the firms in the industry. These quantities may vary significantly over time, yet it is necessary to characterize the equilibrium but for the merger with a single price, aggregate output, and set of shares. That is typically done by averaging historical data over a period such as the most recent twelve months for which the data are available. Averaging over a longer time period may be best if shares are volatile, and averaging over a shorter time period may be best if the data exhibit trends or structural shifts causing the near future to be unlikely to resemble the past of just a year ago. In exceptional cases, there may be sufficient grounds for adjusting historical data to reflect anticipated near-term changes in price or shares but for the merger. For example, one incumbent may be about to bring new capacity on line, or a new firm may be about to enter. As detailed by Werden (2002), the same considerations govern the assignment of market shares when they play the central role in merger analysis.

Unlike the situation with differentiated consumer products, merger simulation in a Cournot industry requires the specification of industry boundaries. Traditional merger analysis supplies them through the delineation of a relevant market, and as
explained by Werden (1998, pp. 384–98), that requires either estimating or intuiting the elasticity of demand for any candidate market: To declare that any group of products in any area constitute a relevant market is to declare that demand for them is no more elastic than some critical value (which depends on the functional form of the demand curve). With the rough estimate of the industry’s elasticity of demand supplied by market delineation, along with a but-for equilibrium, it is straightforward to calibrate a Cournot model, provided that simple functional forms for demand and cost are assumed. The demand elasticity and shares are critical to the simulation, but important predictions, including the percentage change in price and output caused by the merger, are independent of the but-for price and aggregate output.

For example, inverse demand may be of the form \( p = a - bX \), in which \( p \) and \( X \) are the industry price and quantity, and the parameters \( a \) and \( b \) are positive. Values for these parameters are implied by the but-for industry price \( (p_0) \), aggregate output \( (X_0) \), and elasticity of demand \( (\varepsilon_0) \): \( a = p_0(\varepsilon_0 - 1)/\varepsilon_0 \), \( b = p_0/\varepsilon_0X_0 \). An isoelastic demand function can be calibrated similarly. Of course, either the demand elasticity or \( b \) itself can be estimated with suitable data, and if \( b \) were estimated, the calibration would use \( \hat{b} \), the point estimate of \( b \). The calibrated value for \( a \) is then \( p_0 - bX_0 \). If the sample means of the data used to estimate \( b \) are used for \( p_0 \) and \( X_0 \), the calibration uses the standard point estimate of \( a \), but \( p_0 \) and \( X_0 \) typically are not the sample means.

The marginal-cost functions are calibrated using the Cournot equilibrium conditions, which can be written \( c_{i0} = p_0(\varepsilon_0 - s_{i0})/\varepsilon_0 \), with \( c_{i0} \) denoting firm \( i \)'s marginal cost in the but-for equilibrium, and \( s_{i0} \) its output share. If marginal costs are assumed to be invariant to output, this expression gives the marginal costs directly. If, instead, the marginal cost of firm \( i \) is assumed to take the form \( x_i/k_i \), it is straightforward to calculate the \( k_i \)'s from the equilibrium conditions.

As an illustration of Cournot merger simulation, suppose the two largest firms propose to merge in an industry with output shares of .4, .3, .2, and .1, and a demand elasticity of \(-1.5\). The predicted effect of the merger is the difference between the computed post-merger equilibrium and the but-for equilibrium used to calibrate the model. Assuming linear demand, no merger synergies, and marginal costs proportionate to capital stock, it is simple to calculate that the merger would cause
industry price to increase by 5.5%.

Simulating the merger facilitates the examination of sensitivity to the elasticity of demand. If the demand elasticity were −3 and everything else were the same, the predicted price increase would be just 1.8%. Simulating the merger also facilitates the examination of sensitivity to the functional form of demand. With isoelastic demand and a pre-merger elasticity of −1.5, the predicted price increase would be 8.3%. Simulation also makes it fairly simple to compute the effects of the merger on consumer and total surplus.

As noted above, the most realistic marginal-cost assumption may be constant marginal cost up to a capacity constraint. Adding capacity constraints to the simulation can lead to much larger price-increase predictions because non-merging firms may be much less able to expand output following a merger. The analysis by the U.S. Department of Justice in the Georgia-Pacific (2001) case was heavily influenced by tight constraints on the ability of the non-merging firms to expand output. Suppose in the foregoing hypothetical that each firm has constant marginal cost up to a capacity constraint at 110% of its but-for output. With a demand elasticity of −1.5, the merger would cause price to rise 9.0% with linear demand and 18.7% with isoelastic demand. In contrast, without the capacity constraints, the merger would cause price to increase by 5.0% with linear demand and 8.5% with isoelastic demand. While capacity constraints may add critical realism to a merger simulation, they require more complex calculations to compute the post-merger equilibrium.

The Mechanics of Bertrand Merger Simulation. Merger simulation in a Bertrand industry proceeds in the same three steps as merger simulation in a Cournot industry, although the first two steps raise quite different issues. The focus in a Bertrand merger simulation is almost entirely on the demand side of the industry, because the unilateral effects of a merger in a Bertrand industry arise from the internalization of competition among the brands combined by the merger. Moreover, the demand side in a Cournot industry is relatively simple, with only a single demand elasticity at issue, while the demand side with differentiated products can be quite complex. Modeling and estimating demand, or otherwise calibrating it, raise important, and often difficult, issues in a Bertrand industry, which simply do not arise in a Cournot industry.
Furthermore, the simplest cost assumption—constant marginal costs with no capacity constraints—is not problematic in a Bertrand industry, although it is in a Cournot industry. With differentiated consumer products, marginal costs typically are essentially constant throughout the relevant range of output. And while non-merging firms may have capacity constraints, they are unlikely to bind because the outputs of non-merging firms are unlikely to change much following a merger.

Having specified the functional forms of the demand and cost functions, their parameters are then calibrated to make them fit the industry under review. The selection of a set of prices and shares to reflect the equilibrium but for the merger is done much as it is in a Cournot simulation, although the particular demand model chosen may dictate a particular basis for assigning shares in a Bertrand industry. With a choice model, such as logit, shares must be assigned on the basis of a physical unit that serves as a common denominator for alternatives in the choice set, e.g., pounds of bread. Other demand models require that shares be assigned on the basis of expenditures. Unit and revenues shares commonly differ substantially for differentiated consumer products, so attention must be paid to the basis for their assignment.

Another issue in the calibration of a Bertrand simulation is which brands to include. As noted above, the prices of excluded brands are held constant. Since the prices of all substitutes for the merging brands actually increase following a merger in a Bertrand industry, any exclusion of substitutes biases downward the price-increasing effects of the merger. But since the prices of most non-merging brands change little, their exclusion generally imparts only a slight bias. Excluding minor brands and brands thought to be more distant substitutes for the merging brands is useful device for simplifying the calibration of a Bertrand simulation.

Bertrand merger simulations typically are calibrated by estimating the parameters of the assumed demand specification. The point estimates of the relevant slope parameters, e.g., the elasticities themselves with an isoelastic demand system, are used in the simulation. The estimated intercept, or shift, parameters are replaced by values calculated from the prices, shares, and elasticities used as the equilibrium but for the merger. Having calibrated the demand side of the model, no additional information on the cost side is required if marginal costs are assumed to be constant,
as is conventional. Instead, the pre-merger equilibrium conditions are solved for the marginal costs, and the pre-merger marginal costs are assumed to be those in the post-merger equilibrium as well, apart from any cost reductions the merger produces.

Significantly, calibration of a Bertrand simulation does not require estimation. Accounting data on the variable costs associated with the production of branded consumer products commonly provide reasonable estimates of the relevant short-run marginal costs. In some cases, significant conceptual issues may make it difficult to estimate marginal costs, for example, when significant opportunity costs are associated with the use of scarce factors of production with profitable alternative uses. Having calibrated the cost side of a Bertrand simulation, the equilibrium conditions can be solved for the implied demand parameters, provided suitable restrictions are placed on demand. With no restrictions, the $n$ equilibrium conditions could not be solved for $n^2$ demand elasticities, but logit demand, for example, has very few demand parameters. As discussed below, placing restrictions on demand may be desirable even if estimation is used to calibrate the model.

Having calibrated the simulation model, it is a relatively simple matter to incorporate the impact of a merger on the equilibrium conditions and compute the post-merger equilibrium. The merger adds terms to the equilibrium conditions for merging brands relating to the cross-price elasticities of demand between pairs of products that are combined by the merger. The predicted price effects of the merger are the differences between the simulated post-merger prices and shares, and the prices and shares in the but-for equilibrium. If a merger is likely to produce marginal-cost reductions, it is simple to incorporate them.

**The Mechanics of Simulation in Auction Model.** Merger simulation with an auction model is much like that in a Cournot or Bertrand industry, but instead of specifying functional forms for demand and cost, valuation or cost distributions for competing bidders are specified. Athey and Haile (2002) develop techniques for non-parameteric estimation of auction models from bid data, but calibration with minimal data is also possible with certain families of power-related distributions. For those families, the parameters of all of the relevant distributions can be recovered from just the variance of a single valuation or cost distribution in the family. Moreover, that variance can be inferred from data on the a single bidder’s profits (e.g., its cost minus its winning
bid), or estimated from bid data. For these families of power-related distributions, the only other data required for calibration are the winning bid probabilities of the merging firms. The probabilities used to calibrate the model are those expected to prevail but for the merger, and they are based on historical data, just as in other merger simulations.

Consider a procurement auction in which bidders draw costs from a power-related family with base distribution $F(c)$. Because the distributions are power-related, it suffices to calibrate any one of them, and it is convenient to focus on the distribution of the minimum cost, $F_{\min}(c_{\min})$, which is $1 - [1 - F(c)]^r$. Merging bidder $i$’s cost distribution is $1 - [1 - F_{\min}(c_{\min})]^r$ with $r = \pi_i = t_i/t$. Using the $h(\cdot)$ function corresponding to the specified $F(c)$, the variance can be computed from either merging bidder’s winning probability and average profit. If $F_{\min}(c_{\min})$ is uniform on the interval $[a, b]$, the relevant $h(\cdot)$ function is that given above, but with $t = 1$, i.e., $\pi(b - a)/2(2 - \pi)$, which is easily solved for $b - a$, and the variance is $(b - a)^2/12$. The logit model is even simpler, because the variance itself appears in the relevant $h(\cdot)$ function. For example, if a firm with a 50% winning probability has an average profit from winning of 5, the implied standard deviation of the underlying value distribution is easily calculated to be 9.25.

The Oracle case (2004, pp. 1169–70) is the only U.S. merger case in which a merger simulation was introduced at trial. The government’s economic expert, Preston McAfee modeled competition among vendors of highly complex business software as an English procurement auction. The court rejected the predictions of his simulation on the sole grounds that basic inputs into it—the winning bid shares—were “unreliable.”

The “Fit” between the Oligopoly Model and the Industry

General “Fit” Considerations. As argued by Werden, Froeb, and Scheffman (2004), a merger simulation should not be given significant weight by an enforcement agency or court unless the oligopoly model at the heart of the simulation “fits” the industry. Among other things, this means that the model must reflect critical features of the competitive landscape, such as whether the product is homogeneous or highly differentiated. But it does not mean that the model must capture every institutional
detail of an industry.

Models are useful analytical tools because they abstract from the minutiae of real-world complexity. Elaborate attempts to incorporate industry details cause models to lose their value in merger analysis; calibration likely becomes infeasible with available information, and there may no longer be any clear predictions. Fine details of competitor behavior also are unlikely to affect the big picture of how the proposed merger affects competition. Finally, the fine detail of competitor behavior tends to be observed with error by an enforcement agency or a court, so allowance must be made for the possibility of facts that don’t fit because they aren’t true.

The most important test for whether a model fits an industry is whether it explains the past well enough to provide useful predictions of the future. When a merger simulation is used to predict changes in the prices of merging brands, the underlying oligopoly model must explain reasonably well the average level of prices for these brands over a substantial period of time before the merger. However, it is wholly unnecessary for the model to explain week-to-week price movements or special promotions.

As a general matter, the proponent of a merger simulation should be prepared to justify each particular modeling choice on some basis. For most choices, the justification should relate to the factual setting of the industry. For some, an ample justification can be found in an axiom of economics, e.g., that firms maximize profits. For the remaining choices, there should be some sort of sensitivity analysis to indicate whether and how the particular choice matters. Moreover, in evaluating justifications for modeling choices, appreciation is necessary for the artistic nature of economic modeling. Different observers of an industry might reasonably perceive the competitive process differently, and different modelers might reasonably come to different conclusions about how best to capture that process.

**The Fit of Cournot, Auction, and Bargaining Models.** Although oligopolists probably never literally set outputs, the Cournot model may be a reasonably good predictor of the effects of mergers in some industries. A key test is how well it explains the general intensity of competition as reflected in industry margins. The Cournot model predicts that the price-cost margin of each competitor equals its output share divided by the absolute value of the elasticity of demand. It should not
be difficult to determine whether the average level of margins in an industry is roughly as predicted by the Cournot model and whether larger firms have larger margins, also as predicted. Of course, high margins could be observed for a limited time because demand temporarily presses against industry capacity, so support for the Cournot model would require that margins be roughly as predicted by the model for a substantial time. Another useful indication of the utility of the Cournot model may be how well it predicts reactions to entry and exit.

The Cournot model appears not to have been used in any court for the purpose of simulating a merger, but two courts in non-merger antitrust cases have evaluated the fit of Cournot models employed in analogous tasks. The leading example is *Concord Boat* (2000, pp. 1056–57), in which the court held that a symmetric Cournot model was “not grounded in the economic reality” of an industry in which competitors were highly asymmetric. And in *Heary Bros.* (2003, pp. 1066–68), the court held that a “Cournot model does not ‘fit’ the economic reality” of an industry in which highly asymmetric shares were not associated with significant differences in margins. The court, however, unwisely objected to the model on the additional basis that the competitors did not literally choose outputs.

An auction or bargaining model may fit an industry better than the Cournot model if an intermediate good is traded through bidding or negotiations. This is especially true if the good is customized to a significant extent, or otherwise not subject to arbitrage, so prices vary significantly across different transactions. An auction or bargaining model can accurately reflect observed price dispersion in an industry, while the Cournot model cannot. Just as the Cournot model may fit an industry reasonably well even though competitors’ actions are not literally their quantities, an auction model may fit an industry reasonably well even competitors’ actions are not limited to the submission of bids. An auction model is apt to fit better than a bargaining model, even if negotiations follow the submission of bids, when the winner is determined by the bidding process alone, or the party accepting the bids dictates the rules of the game.

An examination of the relationship between winning bids and bidders’ costs or valuations is important in assessing the fit of an auction or bargaining model, much as an examination of margins is useful with a Cournot model. It should be possible
to determine whether bidders’ profits are related to their winning probabilities in the manner predicted by the model. For certain families of power-related distributions, one test involves computing the implied variance separately from data on both merging bidders. If data on costs or valuations are available, it also useful to estimate the variance of the cost or valuation distribution and compare that to the variance inferred from margins. If such data are not available, one may still ask whether the implied variance makes sense. If bidders in a procurement setting appear to have very similar costs in different procurements, an auction model may not be able to explain high observed profits.

A critical issue in evaluating the fit of an auction model relates to the manner in which the merger itself is modeled. The theoretical discussion above modeled a merger as a coalition submitting a single bid after taking the same draws from the cost or valuation distribution that the merging firms would have taken. In some industries, this may be an entirely sensible way to model a merger, for example, because the cost of supplying a customer depends on plant location, and the merged firm has all of the merging firms’ locations. In other industries, this may prove a highly unrealistic assumption. In that event, it may be possible to model mergers differently than has been done in the literature, for example, as just the destruction of one of merging firms. But just as in a Cournot industry, that is not likely to be how a merger really works.

There is little experience in modeling mergers with bargaining models, and little can be said about when a bargaining model in general, and any specific bargaining model in particular, is appropriate. Even if a merger clearly alters bargaining power, analysis predicated on Nash’s axiomatic bargaining solution is not necessarily appropriate because it may not reflect the specific strategic bargaining game being played in the industry. A specific strategic bargaining model likely offers predictions sufficient to provide a sound bases for evaluating how well it explains the past.

**The Fit of the Bertrand Model.** An examination of price-cost margins is critical in the evaluation of the Bertrand model’s fit. Bertrand merger simulations generally infer, rather than measure or estimate, brand-level marginal costs. When that is done, a critical test for the fit of the model is how well the inferred marginal costs match the available evidence on actual marginal costs. If the two match reasonably well, the
Bertrand model, as calibrated, at least reflects the general intensity of competition. Accounting data from the merging firms is likely to provide a reasonable indication of the marginal costs for their brands, and it is important that the model satisfactorily explain the pricing of those brands. If it does not, the reason may be that at least one of the merging brands is not being priced pre merger as the Bertrand model predicts, in which case the model cannot be relied upon to predict post-merger prices. Two other possible reasons that also should be considered are that the functional form of demand has been seriously misspecified, and that demand has been poorly calibrated.

Cost data from non-merging firms is less likely to be available, but it may be apparent without examining any cost data that the inferred marginal costs for one or more non-merging brands are implausible. A negative marginal cost clearly is implausible, yet negative marginal costs sometimes are implied by the Bertrand equilibrium conditions, given the estimated demand elasticities. The inference of a negative marginal cost despite a plausible value for a brand’s own-price elasticity of demand indicates that brand is priced much more aggressively than the Bertrand model predicts. Also of interest is a comparison of the inferred marginal costs across brands. This may reveal implausibly large inter-brand differences. When the Bertrand model cannot explain the pricing of only relatively minor non-merging brands, a simple and satisfactory solution is to drop those brands out of the simulation. When the prices of such brands cannot be satisfactorily modeled, it is best to hold them constant.

Several published merger simulations have tested the fit of the Bertrand model in essentially the foregoing manner. Pinske and Slade (2004) conducted a formal statistical test on the average margin, based on detailed price and cost data, and found that the hypothesis of Bertrand competition could not be rejected. The Bertrand assumption made by Nevo (2000b) was supported by a similar test, albeit with comparatively poor cost data, performed by Nevo (2001).

Other aspects of the fit of the Bertrand model also should be considered. The Bertrand model can be used to predict responses to new product introductions and other such shocks. If there are such events in the available data, it is straightforward to examine the accuracy the model’s predictions. In addition, it is important to
consider whether non-price competition is so important that the predictions of the Bertrand model are likely to be seriously misleading. Finally, in the vast majority of cases in which merging manufacturers of differentiated consumer products do not sell directly to consumers, it is essential to investigate the relationship between the merging manufacturers and retailers.

Published merger simulations typically have modeled consumer products industries as if manufacturers sold directly to consumers, e.g., Nevo (2000b) and Saha and Simon (2000). This is a harmless simplification if retailers apply a constant percentage mark-up to the prices paid to manufacturers, as Werden (2000) found was their practice in the *Interstate Bakeries* (1995) case. Froeb, Tschantz, and Werden (2002) and O’Brien and Shaffer (2003) analyze other scenarios, although they consider only cases involving a monopoly retailer supplied by competing manufacturers. In these models, the effect of a manufacturer merger depends on nature of the possible contracting arrangements between the manufacturers and the retailer, and on the retailer’s freedom to decide which brands to carry.

If non-linear contracts are used, with marginal prices and fixed fees, the merger of the manufacturers might have no effect on retail prices, but it also might have the same effect as if the manufacturers sold directly to consumers. Retail prices may be unaffected by the merger if the retailer is entirely free to choose which brands to carry and therefore can credibly threaten to deal exclusively with one manufacturer. The analysis in that case is essentially that of Bernheim and Winston (1998) and O’Brien and Shaffer (1997). If the retailer carries both merging brands in the pre-merger equilibrium, the merger increases the merging manufacturers’ share of total profits but has no effect on marginal wholesale prices or retail prices, because the same retail prices maximize total profits before and after the merger. Things are quite different with restrictions on the retailer’s freedom to refuse to carry particular brands, because such restrictions give the bargaining power to the manufacturers. In that case, the pre-merger equilibrium has marginal wholesale prices that induce the retail prices the manufacturers would set if they sold directly to consumers, along with fixed fees that transfer all profits to the manufacturers. The merger has the same effects on retail prices as if the manufacturers sold directly to consumers.

If only linear contracts can be used, the double marginalization problem arises.
Competition among manufacturers determines the degree to which they price above their marginal costs, and the retailer acts as a monopolist facing the wholesale prices as its brand-specific marginal costs. The effect of a manufacturer merger is to raise the retailer’s marginal costs, and the effect on retail prices is determined by the curvature properties of retail demand, just as the pass-through of marginal-cost reductions. Besanko, Dubé, and Gupta (2005) find average pass-through rates of about 60%, and rates over 100% are possible. Besanko, Dubé, and Gupta also find wide variation in cross-brand pass through rates, which is not surprising in light of their sensitivity to the idiosyncratic properties of particular functional forms.

The various retailer-manufacturer scenarios imply quite different retail margins. It therefore may be possible to reject one or more of these scenarios empirically. Sudhir (2001) and Villas-Boas (2004) perform empirical analyses along these lines.

**Merger Simulation with Differentiated Consumer Products**

**Illustrative Merger Simulations.** An extensive literature makes quantitative predictions of the unilateral competitive effects of mergers using real-world data. Some of the mergers analyzed are actual proposed mergers, while others are hypothetical mergers used to illustrate the methodology. Some analyses only approximate the post-merger equilibrium and therefore are technically not merger simulations, as that term is defined here. Nearly all of the literature considers differentiated consumer products, and much of it focuses primarily on demand estimation, using a variety of models. An exception is the analysis of Brannman and Froeb (2000), which uses an auction model to simulate hypothetical mergers of spatially differentiated competitors.

The simplest, and least data-intensive, demand model is logit. For purposes of merger simulation, Werden and Froeb (1994, 2002) introduced the ALM (Antitrust Logit Model), a convenient reparameterization of the conventional logit model. The version of the logit model presented above has the “none of the above” alternative in the choice set, making the choice probabilities unconditional. The ALM replaces the unconditional choice probabilities with choice probabilities conditioned on one of the inside goods being chosen; i.e., it replaces them with “shares.” Shares are convenient to work with, while the probability of choosing “none of the above” is not
otherwise considered in merger analysis and poses conceptual difficulties. The ALM treats that choice probability as a scaling factor determined by the aggregate elasticity of demand for the inside goods, $\epsilon$. If $s_i$ is the share of good $i$ and $p$ is the average price of all inside goods, the own-price elasticity of demand for brand $i$ in the ALM is
\[
[\beta p(1 - s_i) + \epsilon s_i]p_i/p,
\]
and the cross-price elasticity of the demand for brand $i$ with respect to the price of brand $j$ is
\[
s_j(\beta p - \epsilon)p_j/p.
\]
Werden (2000) provides a condensed version of his analysis as the U.S. Department of Justice’s expert in the *Interstate Bakeries* (1995) case. In the first application of merger simulation by an enforcement agency to an actual proposed merger, he prepared to testify against the merger partly on the basis of simulations using the ALM. The merging firms were leading U.S. bakers of premium brands of white pan bread, which competed with other premium brands as well as with lower-priced private labels (store brands). In the Los Angeles area, the merging firms were the only significant sellers of premium brands, each accounting for about a third of pounds sold, with private labels accounting for almost all of the rest. In the Chicago area, private labels accounted for almost exactly the same share total pounds sold, but the merging firms faced more competition from other premium brands.

Separately for the Los Angeles and Chicago areas, retail scanner data was used to estimate the aggregate elasticity of demand for premium white pan bread and the logit $\beta$. Since the data reflected retail prices, while the merging firms sold at wholesale, the calibrated prices were the average retail prices minus the 28% margin generally taken by retailers. Table 1 presents the wholesale price increases predicted by the simulation, using the point estimates for the two demand parameters. The first two rows give the average price increases for the merging firms’ premium brands, while the last two rows give average price increases for broader aggregates. The last row includes private label bread, for which prices were held constant in the simulation.

Had the case gone to trial, the merging firms undoubtedly would have argued that competition from private label white breads and competition from breads other than white pan bread would have prevented the merged firm from raising prices.
Both contentions are addressed by merger simulation in a manner likely to be far more reliable than the sorts of impressionistic evidence a court otherwise would have relied upon.

Table 1. Predicted Average Wholesale Price Increases for White Pan Bread in the Chicago and Los Angeles Areas

<table>
<thead>
<tr>
<th>Brand Group</th>
<th>Chicago</th>
<th>Los Angeles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Continental’s Premium White Pan Breads</td>
<td>10.3%</td>
<td>10.0%</td>
</tr>
<tr>
<td>Interstate’s Premium White Pan Breads</td>
<td>5.5%</td>
<td>8.3%</td>
</tr>
<tr>
<td>All Premium Brands of White Pan Bread</td>
<td>6.5%</td>
<td>8.3%</td>
</tr>
<tr>
<td>All Brands of White Pan Bread</td>
<td>3.1%</td>
<td>5.9%</td>
</tr>
</tbody>
</table>

Less restrictive demand models often are preferred to the ALM when abundant data is available, along with abundant time in which to analyze it. However, at the outset of a merger investigation, when little is known about patterns of substitution and no data has been analyzed, the IIA property is the natural assumption. Using the ALM, which is based on the IIA property, a merger can be quickly and cheaply be simulated for a wide range of values for the two demand parameters. These simulations indicate what must be true for the merger to produce significant price increases, assuming Bertrand competition. For example, these simulations indicate whether the merging brands must be especially close substitutes to produce significant price increases. Werden and Froeb (2002b, pp. 74–77) present this type of analysis for the proposed merger of two brewers in Sweden, and Werden, Froeb, and Scheffman (2004) present such an analysis for the WorldCom (2000) case.

The latter merger would have combined the second- and third-largest residential long-distance telecommunication carriers in the United States and was abandoned after a challenge by the U.S. Department of Justice. In the course of the investigation, several approaches to merger simulation, some based on demand estimation, were taken by proponents, opponents, and government agencies, and some are discussed by the ABA (2004, Appendix IA) and Pelcovits (2004). Abstracting from many issues actually confronted in the case, it is interesting to consider the sort of analysis that could have been done during the first week of the investigation, using publically
Academic estimates indicate that demand for residential long-distance service is slightly inelastic, so values of the aggregate elasticity between −1.5 and −0.5 likely include any plausible value. The logit $\beta$ can be calibrated from any brand-level demand elasticity, and WorldCom’s own-price elasticity of demand is used. In keeping with the range of estimates frequently found in empirical studies, values between −4 and −1.25 are considered. For these ranges of elasticities, Figure 1 provides a contour plot of predicted price increases, averaged over the entire industry. The predicted price increases for the merging brands, of course, are substantially greater that the industry average, and the likely price increases for those brands could have been much greater if consumers perceived them as especially close substitutes, contrary to the IIA assumption.

Figure 1. Industry Average Price Increases Predicted from the WorldCom–Sprint Merger

Quantitative analyses of proposed mergers involving differentiated consumer products are commonly performed under the assumption of Bertrand competition, using a choice model related to the ALM. Froeb, Tardiff, and Werden (1996) simulate hypothetical mergers of Japanese long-distance carriers using a logit model that
incorporates brand characteristics. Froeb, Tschantz, and Crooke (2003) simulate the merger of parking lots using a logit model that incorporates travel distance between the parking lots and consumers’ final destinations. Their simulation also accounts for the effects of capacity constraints on optimal prices.


Nevo (2000b) uses the model of Berry, Levinsohn, and Pakes (1995), a mixed-logit model incorporating both brand characteristics and an interaction between them and the random component of utility. He analyzes actual and hypothetical mergers of ready-to-eat cereal producers in the United States, including the merger challenged in the Kraft (1995) case. Gaynor and Vogt (2003) take a similar approach in simulating an actual hospital merger. Similar in spirit is the model of Pinkse and Slade (2004), which constrains the relevant demand elasticities to be functions of closeness metrics defined on brand characteristics. They use their model to simulate mergers of brewers in the United Kingdom.

effects of the merger of leading producers of carbonated beverages. Capps et al. (2003) use the same demand systems to analyze hypothetical mergers of U.S. spaghetti sauce producers.

Merger simulation is also done using the PCAIDS (Proportionately Calibrated AIDS) model, which assumes AIDS demand but calibrates the demand elasticities by assuming the IIA property holds pre merger (making the pre-merger demand elasticities the same as those in the logit model). Epstein and Rubinfeld (2002) propose this model and use it to simulate the mergers in the *Kimberly-Clark* (1995) and *Heinz* (2001) cases. The latter merger involved the second- and third-largest producers of baby food in the United States. It was challenged by the Federal Trade Commission and enjoined on the basis of its likely coordinated effects. Epstein and Rubinfeld (2003) use a version of the PCAIDS model with nests to simulate the hypothetical beer mergers considered by Hausman, Leonard, and Zona (1994).

Baker and Bresnahan (1985) also predict the price effects of hypothetical beer mergers. While akin to merger simulation, their approach differs in critical ways. Most importantly, they do not base their prediction on a one-shot oligopoly model with a Nash, non-cooperative equilibrium. Indeed, the main point of their approach is to estimate best-response functions differing from those derived from the assumption of Nash equilibrium.

**Higher-Order Properties of Demand Specifications.** In simulating a merger involving differentiated consumer products, the main focus generally is on the specification and calibration of demand. Academic work has strongly favored the demand forms best fitting the data and best explaining the observed equilibrium. While those are important considerations, even more important in merger simulation is that any demand function used to model consumer demand has its own particular properties, which substantially affect the price-increase predictions. The magnitudes of the price changes from a merger are determined not just by the own- and cross-price elasticities of demand, but also by how those elasticities change as prices change, and the higher-order, “curvature” properties of demand are preordained by any conventional functional-form assumption.

The impact of functional-form assumptions is most easily seen when there is a single product. Figure 2, taken from Crooke et al. (1999), plots segments of four
conventional functional demand forms calibrated to have a common competitive equilibrium. These demand curves share the same price, quantity, and elasticity of demand (–2) at the right endpoints of the plotted segments, and at that common point, the demand curves intersect an arbitrary constant marginal cost curve. Given this marginal cost, the monopoly prices with each of the four demand curves are the left endpoints of the plotted segments. Functional form for demand may appear even more important in Figure 2 than it really is, because the axes have been translated to enlarge the relevant range of demand. The lowest monopoly price, with linear demand, is 25% greater than the competitive price, while the highest monopoly price, with isoelastic demand, is twice the competitive price.

The implication of the foregoing for merger simulation is immediate: Merger simulations predict relatively low price increases with linear and logit demand functions, in which the own-price elasticities increase relatively rapidly as prices increases. And merger simulations predict relatively high price increases with AIDS and isoelastic demand functions, in which the own-price elasticities increase relatively slowly, or not at all, as prices increases. Crooke et al. (1999) illustrate this by simulating mergers in randomly generated industries. That exercise demonstrates roughly what Figure 2 suggests.
Also interesting is an illustration based on the *WorldCom* (2000) case. Werden, Froeb, and Tschantz (2004) simulated the merger under three different demand assumptions, using elasticity estimates supplied by Jerry Hausman in a submission opposing the merger before the Federal Communications Commission. The second column in Table 2 indicates the huge differences in predicted increases, absent any merger synergies, with the three demand functions. The third through fifth columns indicate the marginal effect, at the post-merger equilibrium absent synergies, of a small reduction in Sprint’s marginal cost. The table displays the effect on the price of the Sprint brand, on the price of the other merging brand, and on the price of the principal rival brand. With linear demand, the price of the Sprint brand would be reduced by half of the amount by which its marginal cost is reduced, while with isoelastic demand, it would be reduced by nearly four times the amount of the marginal-cost reduction.

<table>
<thead>
<tr>
<th>Demand Form</th>
<th>Price Increase</th>
<th>Pass-Through Rate from Sprint Cost to WorldCom</th>
<th>AT&amp;T</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>2.3%</td>
<td>0.502</td>
<td>0.001</td>
</tr>
<tr>
<td>AIDS</td>
<td>13.8%</td>
<td>1.807</td>
<td>0.032</td>
</tr>
<tr>
<td>Isoelastic</td>
<td>16.4%</td>
<td>3.838</td>
<td>-0.343</td>
</tr>
</tbody>
</table>

It is not coincidental that the impact of the marginal-cost reductions on Sprint’s price is roughly proportional to amount of the price increases absent synergies. Werden, Froeb, and Tschantz (2004) demonstrate that the higher-order properties of demand causing larger price increase absent synergies also cause greater pass-through of synergies. And it is notable is that, with isoelastic demand, Sprint’s marginal-cost reductions cause an *increase* in the price of its merger partner. That is true with logit demand as well and is simply an idiosyncratic property of those two demand forms.

While isoelastic demand appears never to have been explicitly assumed in any published merger simulation, it is reasonably common to approximate the post-
merger equilibrium in a manner that effectively assumes both isoelastic demand and that shares are unaffected by the merger. Hausman and Leonard (1997) argue that such an approximation is quite close to the exactly computed post-merger equilibrium, but they assume an AIDS model, which yields predictions reasonably close to those with isoelastic demand, and they consider a merger with small predicted price effects, which minimizes the error of approximation.

**Parameter Parsimony in Demand Specifications.** The number of own- and cross-price elasticities in a differentiated products merger simulation is the square of the number of products, and even a fairly narrow product category may have hundreds of products at the level of UPCs (universal product codes). The number of elasticities can exceed the number of observations, and even if there are several times as many observations as elasticities, imposing structure on demand may be desirable to reduce the variances of the elasticity estimators. The art of demand estimation is achieving necessary and desirable parameter parsimony without sacrificing critical flexibility.

At one extreme in the balance between flexibility and parameter parsimony is the use of a flexible functional form such as an AIDS model (see Pollak and Wales 1992, pp. 60–67), which, in theory, do not constrain the elasticities. Of course, flexible functional forms are only flexible to the “second-order,” i.e., they approximate arbitrary functions at a single point. Their flexibility does not extend to the higher-order properties critical in determining the price effects from a merger. Moreover, White (1980) shows that the particular flexible form assumed affects the elasticity estimates. In practice, the use of flexible functional forms also requires imposing considerable structure on the data. Many separate products UPCs are aggregated up to the “brand level,” which essentially assumes all the aggregated products have the same elasticities. And multi-stage budgeting is assumed in order to further limit the number of brands at the bottom level, although that imposes potentially unrealistic substitution patterns on pairs of brands not together at the bottom level. This imposition of structure on the data is still not sufficient to preclude point estimates inconsistent with strong prior beliefs. Brand-level demands may be estimated to be inelastic, and pairs of obviously substitute brands may be estimated to be complements. Remarkably, Abere et al. (2002) find nothing implausible about their finding of a complementary relationship between Coca-Cola’s brands of carbonated
beverages and the brands of Cadbury Schweppes.

At the other extreme in the balance between flexibility and parameter parsimony is the use of the ALM, in which two parameters determine all of the own-price and cross-price elasticities of demand for the included brands. The imposition of the IIA property is only natural in the absence of any data, but when data, time, and resources allow the closeness of the merging brands to be investigated empirically, it is certainly desirable do so. Imposing the IIA property assumes away both the possibility that the merging brands are especially close substitutes and the possibility that they are especially distant substitutes.

Several less restrictive alternatives to the ALM incorporate brand characteristics in potentially useful ways. In the nested logit model, developed by McFadden (1977) and discussed by Anderson, de Palma, and Thisse (1992, pp. 46–48), brands can be placed together in a nest based on shared attributes, and whether these brands are especially close substitutes can be determined empirically. Similarly, Bresnahan, Stern, and Trajtenberg (1997) specify “principles of differentiation” that distinguish brands, for example, on the basis of whether they have strong customer recognition, and they estimate the importance of sharing the same dimensions of differentiation. Their approach is somewhat more flexible than the nested logit model, but both rely on a priori segmentation of an industry. Such a priori segmentation is not employed by the most popular approach in the academic literature, which is to generalize random utility models by treating the coefficients of the indirect utility function as random variables. This results in a “mixed-logit” model, because the random component of utility, reflecting brand characteristics observable to the consumer but not the econometrician, is assumed to be distributed just as in the logit model.

The best known mixed-logit model, that of Berry, Levinsohn, and Pakes (1995), assumes consumers differ in the weights that they place on individual brand characteristics. The coefficients on characteristics are assumed to be normally distributed over the population of consumers, and a brand’s aggregate demand is obtained by integrating over the population distribution. The estimates of the means and variances of the coefficient distribution are those best fitting the moments of the data. The implications of the normality assumption for merger simulations do not appear to have been investigated, and Nevo (2001) replaces that assumption with the
assumption that those distributions are determined by the distributions of relevant consumer demographics. He exploits cross-sectional demographic variation in census data to estimate those distributions. Like Berry, Levinsohn, and Pakes, Nevo assumes the distributions of individual random coefficients are independent.

While the foregoing analysis is based on aggregated data, McFadden and Train (1999) and Berry, Levinsohn, and Pakes (2004) estimate mixed-logit models with individual data. This allows demographic characteristics at the level of individual consumers to explain their choices. With aggregate data, demand can be estimated on the basis of inter-temporal movements in prices, which could result from various exogenous factors, including changes in the distribution of consumer characteristics. Berry, Levinsohn, and Pakes (1995) exploit changes in the choice set over time to estimate the means of the random coefficients on brand characteristics. With data on the characteristics and choices of individual consumers, demand can be estimated from a single cross section, and Berry, Levinsohn, and Pakes (2004), for example, use survey data on second choices.

Mixed-logit models have potentially significant advantages over the ALM. Because of the more flexible way they model consumer heterogeneity, mixed-logit models permit merging products to be particularly close or distant substitutes, as indicated by the data. They also make it possible to have niche products, with high prices despite low shares. Moreover, as Petrin (2002) illustrates, mixed-logit models may provide more reasonable estimates of the consumer welfare effects of adding or subtracting brands. The logit model assumes that a major component of utility is a random variable with a Type 1 extreme value distribution. One implication is that some consumers derive significant additional utility from switching to a new product even though it differs little from many pre-existing products. If brand characteristics explain choices well, this occurs far less in mixed-logit models.

The distributional assumption for unobserved brand characteristics shared by logit and mixed-logit models may be a source of difficulty. Making many of the assumptions employed to estimate a mixed-logit model from aggregate data, Bajari and Benkard (2004) develop a two-step estimator for a hedonic model without an error term capturing unobserved product characteristics. The first step estimates the hedonic price equation, and the second backs out the parameters of the indirect utility
function from the derivatives of the hedonic price equation and the conditions for utility maximization. Bajari and Benkard find that their approach estimates demand to be more elastic than does a mixed-logit model, which they attribute to limitations of the logit error term.

The field of industrial organization has focused, perhaps excessively, on complex demand forms. Berry (1994) showed that equilibrium conditions can be used to identify a demand model without computing the equilibrium, but computing the equilibrium is central to merger simulation. The mathematical properties that assure existence, uniqueness, and simple computation of equilibrium in the ALM may be lost when complex models are used. It is notable that Nevo (2000b) does not actually compute post-merger equilibria in his analysis of ready-to-eat cereal mergers.

**Estimation Issues with Scanner Data.** The data most often used in merger cases to estimate consumer demand is generated by Information Resources, Inc. and ACNielsen through checkout scanners at the point of sale, primarily in grocery retailers. As detailed by Baron and Lock (1995) and Bucklin and Gupta (1999), both the marketing departments of major corporations, the main customers for the data, and marketing researchers, rely heavily on these data. Although some less aggregated data have been provided to researchers, a merger investigation typically has several years of weekly data on the number of units sold and average revenue aggregated over all sampled stores in metropolitan area. These data are aggregated to the level of “products,” which include many individual UPCs, and they do not reflect price reductions from the use of coupons. IRI and Nielsen also provide data on the use of special promotions, such as the proportion of stores using in-store displays, although that information may not be employed in the estimation of demand elasticities.

As documented by Hosken and Reiffen (2004), grocery prices in the United States vary over time largely due to temporary price reductions (TPRs). In non-public data used to compile the Consumer Price Index, they find that prices for 20 categories of products are at their annual modal value about 60% of the time and at least 10% below their modal value about 15% of the time. They also find that, for the vast majority of categories, 20–50% of the inter-temporal variation in prices results from TPRs. Pesendorfer (2002) finds a similar pattern in daily scanner data for a single
product category—ketchup. He argues, similarly to Sobel (1984), that the observed timing and duration of TPRs is explained by a model in which retailers price discriminate among consumers who sort themselves on the basis of their willingness to postpone purchases until a TPR.

Because so much of the price variation in scanner data arises from TPRs, questions are raised about whether there is any workable way to estimate the demand elasticities relevant to competitors’ post-merger decisions on permanent price changes. Whatever the explanation for TPRs, they clearly have different purposes and effects than permanent price changes. This is true if TPRs are used to price discriminate, or if they are used to induce consumers to try new things in the hope that some will continue to purchase without the TPRs. Nevo and Wolfram (2002) find that coupons have the latter effect. TPRs also are commonly accompanied by in-store promotions and advertising designed to produce transitory shifts in demand. If TPRs work at all as intended, the quantity response to a TPR should be greater than the quantity response to a permanent price change.

Even if TPRs are random events associated with mixed strategy equilibria, as suggested by Varian (1980), they pose a challenge in estimation because they allow thrifty consumers to stockpile storable products during TPRs and draw down inventories during periods of regular prices. Indeed, Van Heerde et al. (2004) estimate that about a third of the quantity increases from a TPR stem from the time shifting of purchases by consumers. As Hendel and Nevo (2001) demonstrate, failing to account for consumer inventory behavior can lead to large errors in estimating the relevant demand elasticities. An ad hoc solution is to aggregate the weekly data up four-week periods, but this entirely eliminates the problem only in the extraordinary case of consumer inventories varying over precisely the same four-week cycle. And even in that extraordinary case, aggregating to four-week periods may eliminate most of the price and quantity variation in the data and also greatly increase the standard errors of the estimated coefficients. Erdem, Imai, and Keane (2003), Hendel and Nevo (2003), Sun, Neslin, and Srinivasan (2003), and Romeo and Sullivan (2004) all make efforts to incorporate inventories into econometric models of consumer demand. Of course, that is unnecessary for products likely to have minimal inventoring, particularly highly perishable food items.
As generally provided by IRI and Nielsen, scanner data are aggregated over products, stores, and time. As noted by Hosken et al. (2002) and the ABA (2004, Appendix), all three types of aggregation potentially create estimation problems because the prices in the aggregation are far from perfectly correlated. Although TPRs are commonly sponsored by the manufacturers, Chevalier, Kashyap, and Rossi (2003) find that retailers reduce their margins when they reduce their prices, and different retailers may take manufacturer-sponsored TPRs to different degrees, at different times, and accompany them with different sorts of in-store and advertising promotions. Pesendorfer (2002) finds a correlation of prices for Heinz ketchup across stores within a chain of only about 0.5 and a correlation across chains near zero. Consequently, average revenue for a given product in a given week may reflect purchases made by different consumers facing significantly different relative prices.

For the foregoing reasons, scanner data inherently present aggregation issues (see generally Stoker 1993). The fundamental problem is that scanner data employ a particular price index, average revenue, which is unlikely to be the correct price index. Consider, for example, the case of $n$ identical stores with identical demands of the form $x_i = a - b p_i$. Aggregate demand per store is $a - b \sum p_i / n$, but the price index in the scanner data is $\sum p_i x_i / \sum x_i$ rather than $\sum p_i / n$. Regressing quantity on the scanner data price index, therefore, yields a biased estimate of $b$. The actual aggregation problem is obviously far more complex than this, and it not so clear what the impact aggregation really is. In practice, scanner data are further aggregated to the “brand level,” for example by combining package sizes and flavors, and a single “brand” may consist of hundreds of UPCs. But this poses no real difficulty because an appropriate price index can be used.

That scanner data reflect retail prices, while mergers generally involve manufacturers selling at wholesale prices, presents the serious challenges discussed above concerning the impact of different contractual relationships. But if those challenges, and the problems presented by demand estimation, can be overcome, observing retail rather than wholesale prices is more of a blessing than a curse. The demand of final consumers is fundamentally what drives wholesale pricing, and retail data may be a far richer source of information. Wholesale price data for competing manufacturers can have so little independent variation that they are
worthless in estimating demand. As noted above, manufacturers rely on retail data to inform pricing and other marketing decisions. Merger investigations should be able to rely on the same data for similar purposes.

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