

## A robust test for consumer welfare enhancing mergers among sellers of a homogeneous product

Luke M. Froeb<sup>a</sup>, Gregory J. Werden<sup>b,\*</sup>,<sup>1</sup>

<sup>a</sup>*Owen Graduate School of Management, Vanderbilt University, Nashville, TN 37203, USA*

<sup>b</sup>*Antitrust Division, US Department of Justice, Washington, DC 20530, USA*

Received 3 June 1997; accepted 22 October 1997

---

### Abstract

Antitrust enforcement agencies and courts use net effect on price as a touchstone for the legality of mergers. This paper derives a simple condition for implementing that standard when industry equilibrium is static Nash in quantities (Cournot), and that condition is robust to different specifications of demand and cost. © 1998 Elsevier Science S.A.

*Keywords:* Antitrust; Mergers; Cournot

*JEL classification:* L41; D43

---

Consider a homogeneous product industry in which the competitive interaction is Nash in quantities (Cournot). With neither entry nor efficiencies, Farrell and Shapiro (1990) demonstrated that a merger causes the merged firm to reduce output, and although non-merging firms increase output in response, total industry output falls and price increases. If, however, a merger also reduces the merging firms' marginal costs, the cost reduction tends to offset the anticompetitive effect of the merger on prices. If the merger reduces the marginal costs of the merging firms by a sufficient amount, it increases industry output and decreases industry price.

Following Williamson (1968), economists generally have favored total economic welfare as the touchstone for antitrust policy toward mergers; however, antitrust enforcement authorities have argued, and courts have agreed, that a merger should be deemed unlawful if its likely effect is to increase prices, i.e., to diminish consumer welfare.<sup>2</sup> We provide a simple calculation determining the marginal cost reduction for the merging firms necessary and sufficient to offset the incentive to restrict output, and hence, the marginal cost reduction necessary and sufficient to prevent a diminution in consumer welfare.<sup>3</sup>

\*Corresponding author. Tel.: +1 202 3076366; fax: +1 202 3073372; e-mail: gregory.werden@usdoj.gov

<sup>1</sup>The views expressed herein are not purported to reflect those of the U.S. Department of Justice.

<sup>2</sup>See *FTC v. University Health, Inc.*, 938 F.2d 1206, 1222–1223 (11th Cir. 1991); *United States v. United Tote, Inc.*, 768 F. Supp. 1064, 1084–1085 (D. Del. 1991).

<sup>3</sup>Werden (1996) showed that, in a Bertrand model with differentiated products, the marginal cost reductions necessary to restore the pre-merger prices can be calculated without making any assumption about demand, and also provided a formula for that calculation.

Crucially, the analysis holds whenever a unique Cournot equilibrium exists. Robustness to the cost assumption is significant with Cournot models because the literature on Cournot mergers has noted sensitivity to the shape of the marginal cost curve and debated the appropriate assumption (see Perry and Porter, 1985; Werden, 1991).

Define

$q_i$  = the quantity of firm  $i$

$c_i$  = the marginal cost of production for firm  $i$

$Q$  = aggregate industry quantity

$s_i$  = the output share of firm  $i = q_i/Q$

$p$  = industry price

$\epsilon$  = the equilibrium elasticity of industry demand, defined to be positive

The familiar first-order condition for profit maximization is

$$\frac{p - c_i}{p} = \frac{s_i}{\epsilon}. \quad (1)$$

This first-order condition holds both pre- and post-merger, and a merger that does not change  $p$  also must not change  $Q$  or  $\epsilon$ . Thus, the first-order conditions for the non-merging firms are satisfied post-merger by the  $q_i$  that satisfy them pre-merger. With both  $Q$  and the  $q_i$  for the non-merging firms unchanged by a merger, the post-merger output share of merged firm must equal the sum of the pre-merger output shares of the merging firms.

Since our focus is on marginal cost, we rewrite the first-order condition as

$$c_i = (\epsilon - s_i)p/\epsilon, \quad (2)$$

which holds for the merged firm as well. Letting the merging firms be  $j$  and  $k$ , and substituting the sum of their shares for the merged firm's share, yields  $(\epsilon - s_j - s_k)p/\epsilon$  for the merged firm's marginal cost in the post-merger equilibrium. The pre-merger, share-weighted average marginal cost for the merging firms is

$$\frac{p[s_j(\epsilon - s_j) + s_k(\epsilon - s_k)]}{\epsilon(s_j + s_k)}. \quad (3)$$

Consequently, the proportionate reduction in marginal cost necessary to restore the pre-merger price is

$$\frac{2s_j s_k}{\epsilon(s_j + s_k) - (s_j^2 + s_k^2)}, \quad (4)$$

which, in the symmetric case ( $s_j = s_k = s$ ), simplifies to

$$\frac{s}{\epsilon - s}. \quad (5)$$

The Horizontal Merger Guidelines promulgated by the U.S. Department of Justice and the Federal

Table 1  
 Percentage marginal cost reduction that restores the pre-merger price

$\Delta\text{HHI}$	Elasticity of demand		
	1	2	3
100	7.6	3.7	2.4
500	18.8	8.6	5.6
1000	28.8	12.6	8.1
2500	54.7	21.5	13.4
5000	100.0	33.3	20.0

Trade Commission summarize the shares of the two merging firms by the “change in the HHI” ( $\Delta\text{HHI}$ ).  $\Delta\text{HHI}$  is defined as twice the product of the shares of the merging firms, with the shares expressed in percentage terms. Table 1 above provides illustrative calculations based on Eq. (5) for plausible elasticities of demand and various  $\Delta\text{HHI}$ s.<sup>4</sup> The table indicates: (1) the necessary marginal cost reductions are quite sensitive to the elasticity of demand, (2) modest marginal cost reductions (e.g. 5%) prevent price increases following mergers of modest size (e.g.,  $\Delta\text{HHI}=100$ ), and (3) implausibly large cost reductions (e.g. 20%) may be necessary to prevent very large mergers (e.g.,  $\Delta\text{HHI}=2500$ ) from raising price.

## References

- Farrell, J., Shapiro, C., 1990. Horizontal mergers: an equilibrium analysis. *American Economic Review* 80, 107–126.
- Perry, M.K., Porter, R.H., 1985. Oligopoly and the incentive for horizontal merger. *American Economic Review* 75, 219–227.
- Williamson, O.E., 1968. Economies as an antitrust defense: the welfare tradeoffs. *American Economic Review* 58, 1372–1376.
- Werden, G.J., 1991. Horizontal mergers: comment. *American Economic Review* 81, 1002–1006.
- Werden, G.J., 1996. A robust test for consumer welfare enhancing mergers among sellers of differentiated products. *Journal of Industrial Economics* 44, 409–413.

<sup>4</sup> $\Delta\text{HHI}$  predicts rather well; substituting the geometric mean of the merging firms’ shares,  $(s_j s_k)^{1/2}$ , for  $s$  in Eq. (5) provides a good approximation of Eq. (4).