THE ANTITRUST LOGIT MODEL FOR PREDICTING UNILATERAL COMPETITIVE EFFECTS

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We briefly present the Antitrust Logit Model (ALM)[1] and explain the constructive role it can play in the analysis of unilateral competitive effects from mergers in differentiated consumer products industries.[2] At a minimum, we believe that the ALM provides a valuable quick look at potential price increases.

The ALM is a logit model, which is a particular random utility choice model. In that broad class of models, each consumer makes a single choice from a "choice set," generally consisting of certain "inside goods" and an "outside good," reflecting the choice of "none of the above." While similar in concept, the inside goods may be more or less inclusive than a relevant antitrust market. Because consumers make a single choice, each good is a substitute for all others in the choice set.

Consumers make choices based on the utilities associated with alternatives. In the simplest models, there is a non-random component of utility, $\alpha_i - \beta p_i$, where α_i is a constant indicating, roughly, good i's average brand preference, and p_i is its price. The price coefficient, β, determines the

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[1] For details, see Gregory J. Werden & Luke M. Froeb, *Simulation as an Alternative to Structural Merger Policy in Differentiated Products Industries*, *in* THE ECONOMICS OF THE ANTITRUST PROCESS 65 (Malcolm B. Coate & Andrew N. Kleit eds., 1996); Gregory J. Werden, Luke M. Froeb & Timothy J. Tardiff, *The Use of the Logit Model in Applied Industrial Organization*, 3 INT'L J. ECON. BUS. 83 (1996); Gregory J. Werden & Luke M. Froeb, *The Effects of Mergers in Differentiated Products Industries: Logit Demand and Merger Policy*, 10 J.L. ECON. & ORG. 407 (1994).

[2] For an introduction to merger simulation, see Gregory J. Werden, *Simulating Unilateral Competitive Effects from Differentiated Products Mergers*, ANTITRUST, Spring 1997, at 27; Gregory J. Werden, *Simulating the Effects of Differentiated Products Mergers: A Practical Alternative to Structural Merger Policy*, 5 GEO. MASON L. REV. 363 (1997).

elasticities of demand.[3] These models also have a random component, which is a device for incorporating heterogeneity in consumer preferences. Different assumptions about the statistical distribution of that component produce different models, and one convenient assumption produces the logit model. The logit model takes its name from the fact that the S-shaped "logistic" function describes the choice probabilities for the entire population of consumers.

Simple logit models exhibit the Independence of Irrelevant Alternatives (IIA) property. This means that substitution is proportionate to relative shares. For example, if goods *A* and *B* have shares of 30 percent and 10 percent, an increase in the price of any other good in the choice set necessarily induces three times as much substitution to *A* as it does to *B*. We find the IIA property to be a useful way to define what it means for all goods to be equally close substitutes for each other. The property implies that an increase in the price of an inside good induces an equal proportionate increase in the quantity demanded for all other inside goods, i.e., that all of these cross elasticities of demand are equal.

The ALM is a user-friendly version of logit model for antitrust practitioners, formulated in terms of "shares" within the set of inside goods. These are much like market shares, but the inside goods need not be the relevant market. The probability of choosing the outside good is neither otherwise considered in merger antitrust nor free of conceptual difficulties. The ALM treats that probability as just a scaling factor determined by the aggregate elasticity of demand for the inside goods, denoted ε. If this elasticity is sufficiently large, the availability of outside choices prevents mergers of inside goods from significantly increasing consumer prices. In this sense, ε serves basically the same purpose as market delineation. Given the value of ε, the price coefficient, β, determines the responsiveness of choices to changes in prices: The greater the value of β, the greater the substitutability among the inside goods.[4]

To predict price increases from a merger using the ALM, one needs prices and shares, which are routinely determined in merger investigations, and values for ε and β. Although estimating ε can be challenging, that challenge is already present in traditional antitrust analysis because

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[3] If the choice probability for good i is π_i, the elasticity of demand for good i is $\beta p_i(1 - \pi_i)$, and the cross elasticity of the demand for good i with respect to the price of good j is $\beta p_j \pi_j$. This cross elasticity is misstated by Philip Nelson & Su Sun, *Consumer Savings from Merger Enforcement: A Review of the Antitrust Agencies' Estimates*, 69 ANTITRUST L.J. 921, 956, 959 (2002).

[4] Denoting the share of good i as s_i and the average price of all inside goods as p, the own elasticity of demand for good i is $[\beta p(1 - s_i) + \varepsilon s_i]p_i/p$, and the cross elasticity of the demand for good i with respect to the price of good j is $s_j(\beta p - \varepsilon)p_j/p$.

the process of market delineation requires at least intuiting the value of ε from non-quantitative evidence.[5] The value of β can be estimated from aggregate data on prices and quantities of actual transactions, household-level data on actual choices, or survey data; and it can be inferred in several ways. This inference can be made from observed patterns of diversion resulting from a natural experiment, such as the addition or subtraction of a brand. And under the conventional assumption that observed prices and shares are the product of a non-cooperative equilibrium in which firms compete on the basis of price, the value of β (given that of ε) can be inferred from the price-cost margin of any major brand.

Although economists have long noted that the IIA property is not likely to hold in the real world, we still think that the ALM is very useful. It is at least a good starting point in the analysis of differentiated products mergers. Until reliable contrary evidence is uncovered, we think it sensible to presume that the products of the merging firms are neither especially close nor especially distant substitutes, which means that the IIA property holds approximately.[6] Before it is possible to begin the process of estimating the relevant demand elasticities, a few minutes work with the ALM provides an initial indication of the potential consumer injury from a differentiated products merger. The ALM, thus, serves as a screen comparable to that provided by market shares in traditional antitrust analysis, but the ALM is a vast improvement because it offers quantitative price-increase predictions.[7] Moreover, the ALM also indicates the implications of certain facts further investigation may reveal, e.g., whether substantial price increases require that the merging brands be particularly close substitutes.

With an abundance of time and data exhibiting just the right kinds of price and quantity variation, a host of models may be used with some success to estimate demand and predict the price effects of differentiated products mergers. Nevertheless, the simple logit model and various generalizations of it still have much to offer. There is much to be said for

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[5] *See* Gregory J. Werden, *Demand Elasticities in Antitrust Analysis*, 66 ANTITRUST L.J. 363, 378–91 (1998). Useful thought experiments are suggested by the relationship between market delineation and ε. One might assume that a collection of goods constitutes a relevant market, compute the highest demand elasticity consistent with that assumption, and use that as ε.

[6] Whether the IIA property holds between the merging and non-merging brands tends to be relatively unimportant to the price effects of a merger.

[7] We have used the IIA assumption to calibrate other models, e.g., linear and AIDS. *See* Philip Crooke, Luke M. Froeb, Steven Tschantz & Gregory J. Werden, *The Effects of Assumed Demand Form on Simulated Postmerger Equilibria*, 15 REV. INDUS. ORG. 205 (1999). Using an AIDS model calibrated by assuming IIA was recently advocated for the sort of

letting data speak for itself, but the available data tend not to be rich enough to identify accurately the large number of separate substitution relationships being estimated. Unless restrictions are imposed on substitution relationships, cross elasticities, which are critical to the magnitude of merger price effects, tend to be imprecisely estimated. Estimates often indicate that certain pairs of products are complements when they are known to be substitutes. Strategies for addressing this problem include using the ALM, thus imposing the IIA property, and using various generalizations of the logit model.[8] The latter models relax the IIA property in various ways while still imposing a great deal of structure of substitution relationships.

It is also important to understand that none of the models used to estimate demand elasticities totally lets the data speak for itself. Even models that do not restrict the values of the demand elasticities at a given set of prices and quantities do restrict how those elasticities vary as prices change. Consequently, both the predicted price effects of mergers and the pass-through rates of merger efficiencies are determined to a significant extent by the functional form assumed. Among the models that have been analyzed, linear and logit demand yield the smallest price increases[9] and the smallest pass-through rates for marginal cost reductions.[10] We prefer models yielding relatively small price increases unless evidence indicates that the variation of elasticities as prices change is more realistic in models yielding relatively large price increases.[11]

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quick-look analysis we describe here. Roy J. Epstein & Daniel L. Rubinfeld, *Merger Simulation: A Simplified Approach with New Applications*, 69 ANTITRUST L.J. 833 (2002).

[8] *See, e.g.*, Steven T. Berry, James Levinsohn & Ariel Pakes, *Automobile Prices in Market Equilibrium*, 63 ECONOMETRICA 841 (1995); Timothy F. Bresnahan, Scott Stern & Manuel Trajtenberg, *Market Segmentation and the Sources of Rents from Innovation: Personal Computers in the Late 1980s*, 28 RAND J. ECON. S17 (1997); Aviv Nevo, *A Practitioner's Guide to Estimation of Random-Coefficients Logit Models of Demand*, 9 J. ECON. & MGMT. STRATEGY 513 (2000).

[9] *See* Crooke et al., *supra* note 7.

[10] *See* Gregory Werden, Luke M. Froeb & Steven Tschantz, The Effects of Merger Synergies on Consumers of Differentiated Products (unpublished paper 2001).

[11] Epstein & Rubinfeld, *supra* note 7, at 898 & n.40, opine that the "curvature" properties of the AIDS model are likely to be realistic and cite one empirical article as evidence. We do not think that article addresses the curvature issue.