Algorithms for Full Implementation of the Hypothetical Monopolist Paradigm

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*The views expressed herein are not purported to reflect those of the U.S. Department of Justice.
Goods and Firms

- Anything a firm may sell to a buyer is a good.

- Each good is characterized by its traits, and any difference in traits distinguishes one good from another.

- The relevant market is delineated for the focal good, $g^0$.

- Existing substitutes for $g^0$ are indexed $g_i^0$. 
“A market definition should normally contain two dimensions: a product and a geographic area.” § 2.10

The temporal dimensions of goods also may be a basis on which they are distinguished in market delineation. §§ 5.1–5.3

The market delineation process begins with, and is centered on, individual products of the merging firms. §§ 2.8, 5.17

The market delineation process begins “by looking at a relatively narrow” group of products and area. §§ 3.1, 4.3
OFT Substantive Assessment Reference

The starting point for the iterative process of market delineation is “the product or products immediately affected by the merger.” § 3.18
Important Implications

- Markets exist in the space of traits of goods, not firms.

- The only good assumed at the outset to be in the relevant market for the focal good is the focal good itself.

- Competing goods may have different relevant markets.

- Market delineation concerns demand-side responses of buyers.

- Market delineation alone is not a sufficient basis for assigning market shares.
A market is “a product or group of products and a geographic area in which it is produced or sold.” § 1.0

Market delineation “focuses solely on demand substitution factors—i.e., possible consumer responses.” § 1.0

The “Identification of Firms that Participate in the Relevant Market” is separately addressed in § 1.3.
OFT Market Definition References

“[S]upply-side substitutes should also be included in the market.” §§ 2.11, 3.13

And “market shares [reflect the] set of products whose suppliers would switch.” § 3.21

§ 3.18 suggests that the only alternative approach is “leaving supply-side issues to the analysis of new entry.”

However, the OFT’s Analytical Framework paper for the ICN based market delineation on just demand substitution.

Complements are included “when competition to supply one product constrains the prices charged for the other.” § 5.4
Ranking Substitutes

The $g_i$ are ranked according to their closeness to $g_0$ using a substitute closeness metric $c(g_i|g^0) = c_i$

The $g_i$ as ranked together with $g^0$ form the next-best substitute sequence $\text{nbs}^0 = \langle g^0, g^1, g^2, \ldots, g^I \rangle$

It is used to form the candidate market sequence $\text{CMS}^0 = \langle C^0, C^1, C^2, \ldots \rangle$

$C^j = \{g^0, g^1, g^2, \ldots, g^j\}$
Equally Close Substitutes

If $c_i = c_j$ for any $i \neq j$, $\text{nbs}^0$ is not a unique, but a unique $\text{CMS}^0$ is constructed by grouping all of the $g_i$ that are equally close to $g^0$.

Example:

```
| g_{-3} | g_{-2} | g_{-1} | g^0 | g_1 | g_2 | g_3 |
```

Otherwise identical goods are uniformly spaced on a line as shown. The symmetric pairs $\{g_i, g_{-i}\}$ are grouped.

$\text{CMS}^0$ is $C^0 = \{g^0\}$, $C^1 = \{g^0, g_1, g_{-1}\}$, and so forth.
OFT Market Definition References

§§ 2.9, 3.3, and 4.3 describe an iterative process of market delineation but refer generally to “substitutes” and “areas” without any mention of the concept of a next-best substitute.

OFT Substantive Assessment Reference

§ 3.18 does not refer to a next-best substitute but may be understood to add one product at a time.
Closeness to the Whole Candidate Market

$g_1$ is closest to $g^0$

$g_2$ is closest to $\{g^0, g_1\}$

$g_3$ is closest to $\{g^0, g_1, g_2\}$

$g_{-2}$ is not next-best until all the $g_1, g_2, \ldots$ have been included
Closeness to the Previous Element in nbs$^0$

$g_a$ is closest to $g^0$

$g_1$ is closest to $g_a$

$g_2$ is closest to $g_1$

$g_{-1}$ et al. are not next-best until all the $g_1, g_2, \ldots$ have been included
Substitute Closeness Metrics

- Euclidean distance, travel cost, shipping cost, or a linear combination of differences in traits between $g^0$ and the $g_i$
- Decreasing functions of $\epsilon_{i0}$, the \textit{cross elasticities of demand} for the $g_i$ with respect to the price of $g^0$
- Decreasing functions of \textit{absolute diversions}, e.g., $\epsilon_{i0}q_i$
- Decreasing functions of \textit{diversion ratios}, e.g., $-\epsilon_{i0}q_i/\epsilon_{00}q_0$
- Decreasing functions of \textit{relative diversion ratios}, e.g.,

$$-\epsilon_{i0}q_is_0/\epsilon_{00}q_0s_i$$
**Key Conceptual Issue In Ranking**

Are products A and B below next-best substitutes for each other?

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
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<tr>
<td><strong>Quantities</strong></td>
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<td>(columns=from rows=to)</td>
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# Hypothetical Ranking Data

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<thead>
<tr>
<th></th>
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<tbody>
<tr>
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<td>.07</td>
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<tr>
<td>C</td>
<td>.11</td>
<td>.30</td>
<td>−2.50</td>
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<td>.18</td>
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<tr>
<td>D</td>
<td>.08</td>
<td>.10</td>
<td>.20</td>
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<td>1.00</td>
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<tr>
<td>E</td>
<td>.07</td>
<td>.08</td>
<td>.18</td>
<td>.80</td>
<td>−4.00</td>
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</table>
Rankings based on decreasing functions of cross-elasticities, absolute diversions, and relative diversion ratios:

|  A,  B,  C,  D,  E |
|  B,  A,  C,  D,  E |
|  C,  B,  A,  D,  E |
|  D,  E,  C,  B,  A |
|  E,  D,  C,  B,  A |

Rankings based on decreasing functions of either quantity or revenue diversion ratios:

|  A,  B,  C,  D,  E |
|  B,  A,  C,  D,  E |
|  C,  A,  B,  D,  E |
|  D,  E,  C,  B,  A |
|  E,  D,  C,  B,  A |
The Hypothetical Monopolist Test

Define a *price-increase norm* \( \| p_j^A \| \)

Alternative 1: \( \| p_j^A \| = (p_0^* - p_0^0)/p_0^0 \)

Alternative 2: \( \| p_j^A \| = \sum_i s_i (p_i^* - p_i^0)/p_i^0 \)

Define a price increase *significance threshold* \( t \) (e.g., 0.05)

Each \( C_j \) in CMS\(^0\) is a *market* iff the corresponding \( \| p_j^A \| \geq t \)

The subsequence of CMS\(^0\) for which \( \| p_j^A \| \geq t \) is the *market sequence* \( MS_t^0 = \langle M_s, M_{s+1}, \ldots, M_{L-1}, M_L \rangle \)
“Common practice in both Europe and the US is to consider a price [increase of 5–10 per cent]. This will normally be the Director General’s approach [although the] 5–10 per cent test is a rough guide rather than a rule.” § 3.2

“Arguably, the test should be applied to the value added by the undertaking, not its selling price. In practice, selling prices are more easily seen and are usually the price which will be considered. This is one reason why some flexibility in the size of price differential considered must be retained.” § 3.2 n.7
The Benchmark Prices

The benchmark prices, $p_i^0$, are those likely to prevail in the near future but for the merger, normally the pre-merger prices.

The use of competitive benchmark prices implies that the focal good is a market unto itself, and thus makes the hypothetical monopolist paradigm worthless for merger analysis, if the focal good’s but for price significantly exceeds the competitive level.
OFT *Substantive Assessment* Reference

Benchmark prices “generally” are “prevailing” prices but “other than prevailing prices” may be used “for example where future market prices can be accurately predicted on the basis of, say, changes in an industry’s price regulation.” § 3.20
The Hypothetical Monopolist’s Prices

The *hypothetical monopolist’s prices*, $p_i^*$, are those that would be set by a profit-maximizing monopolist over just the candidate market.

Uniformity of price increases within the candidate market is neither assumed nor typically profit-maximizing.

Critical loss and critical elasticity analyses wrongly assume the hypothetical monopolist increases all prices proportionately.
OFT Market Definition Reference

The relevant prices for the hypothetical monopolist are those that “would maximise its profits.” § 2.8

OFT Substantive Assessment Reference

The issue as “whether it would be profitable for a hypothetical monopolist to impose a” significant price increase. § 3.18
Pricing Outside the Candidate Market

The hypothetical monopolist prices depend on pricing responses for substitutes outside the candidate market.

The simplest assumption is that goods outside the hypothetical monopoly are elastically supplied at prevailing prices.

Outside goods may be assumed to be supplied competitively, if not perfectly elastically, or to interact strategically with inside goods.
The Relevant Market Rule

The *relevant market set*, $M_R$, is selected from $MS_t^0$ with a

*relevant market rule* $MS_t^0 \to M_R$

The strong-form *smallest market principle* holds that $M_R = M_S$

Weaker forms hold that $M_R$ is $M_S$ or some $M_k$ close to $M_S$ in $MS_t^0$

defined by a break in the chain of substitutes, or

for which $\|p_j^{A}\|$ is substantially greater than for $M_S$

Flexibility in the choice of $t$ creates flexibility in $M_S$ and $M_R$
“The Director General would usually use the narrowest potential market definition.” § 2.9
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B,   A,   C,   D,   E
C,   B,   A,   D,   E
D,   E,   C,   B,   A
E,   D,   C,   B,   A

Rankings based on decreasing functions of either quantity or revenue diversion ratios:

A,   B,   C,   D,   E
B,   A,   C,   D,   E
C,   A,   B,   D,   E
D,   E,   C,   B,   A
E,   D,   C,   B,   A
## Percentage Price Increases With Linear Demand and Constant Marginal Cost

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<th>C(2)</th>
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<tr>
<td>2</td>
<td>5.3</td>
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<td>8.1</td>
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<td>12.7</td>
<td>13.9</td>
<td>12.7</td>
<td>17.0</td>
<td>17.0</td>
</tr>
</tbody>
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The Relevant Market Property

The relevant market alleged in a merger complaint is the relevant market property $\mathcal{P}_R$ employing traits (occasionally in a complex manner) to distinguish the goods in the relevant market from all others.

$\mathcal{M}_R$ typically is consistent with more than one $\mathcal{P}_R$.

The selection of $\mathcal{P}_R$ should address which traits really matter to consumers and which new goods should be deemed to be in the relevant market.
Relevant Market Property Example

Suppose $g_0$ is crushed limestone at a point near Columbus, Ohio, and $M_R$ is crushed limestone at a list of nearby quarries.

If the relevant market property were \{crushed limestone quarried in specified central Ohio counties\}, a new limestone quarry in those counties would produce a good in the relevant market, and no granite quarry would produce a good in the relevant market.

A new crushed granite quarry in the specified counties would produce a good in the relevant market if the property were instead \{crushed stone quarried in specified central Ohio counties\}. 
Natural Market Boundaries

*Natural market boundaries* are groupings recognized in the industry or by data compilers and national or other political boundaries.

Natural market boundaries sometimes correspond to breaks in the chain of substitutes.

Using natural market boundaries reduces the number of words in a relevant market property, which is both convenient and reduces the appearance of unreasonable and arbitrary narrowness.
OFT Market Definition References

“[T]he market may be defined to be larger than the narrowest theoretically possible market” when “disaggregated data” for the narrowest market are unavailable. § 2.9 n.5

Relevant market boundaries may and may not be drawn at “gap[s] in the chain of substitution.” §§ 3.9–3.12
Price Discrimination Markets

The foregoing is reapplied to buyer groups for which it is possible for the hypothetical monopolist to charge a separate price and profitable to do so, yielding relevant price discrimination markets.

A price discrimination market set consists of a set of goods with a corresponding set of their buyers.

A price discrimination market property consists of a reference to traits of goods and a reference to a class of buyers.

The algorithms are applied jointly to buyer groups linked by arbitrage constraints.
OFT Market Definition References

§§ 3.7, 3.8 refer to the possibility of price discrimination against “captive customers” and suggest that price discrimination markets are delineated when it is feasible to do so.
Aggregation as a Matter of Convenience

Relevant markets may be aggregated as a matter of convenience if competitive conditions are identical, or nearly so.

- Aggregation is common with good substitutes in supply for some of all of the goods in a relevant market (e.g., hospital services).
- Aggregation is common when relevant markets have a temporal dimension (e.g., electric power).
- Aggregation is common with price discrimination markets, especially if each buyers is in a separate relevant market.