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MGT 439: Real estate finance and capital markets

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Course roadmap

- How is real estate financed?
 - Mortgage securities
 - Mortgage pricing in theory
 - Mortgage pricing in practice
 - Risks affecting mortgage securities
 - Other
- The capital markets for real estate financing
 - Real Estate Securitization
 - Mortgage-backed Securities
 - The subprime 'fiasco'
 - Real estate derivatives (futures, swaps)
- Real estate and the macro economy
- Cool alternatives to DCF real estate valuation
 - Real options


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Preliminaries

- Please read syllabus
 - Text: Brueggeman and Fisher, "Real Estate Finance and Investments", 13th Edition, McGraw-Hill/Irwin.
 - Weekly quizzes (on Blackboard)
 - These must be done individually
 - 30% of final grade
 - Two cases
 - 35% each
 - Group work (4-5 people per group)
 - Re-grading policy
 - Office hours and accessibility


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How is real estate financed


- Mortgages
 - By far the most popular method
 - A debt note is issued and the real asset is pledged against it
 - The terms of payment can be customized
 - Recourse vs. non-recourse
- Other forms of financing
 - Mezzanine debt
 - Junior debt secured by the equity of borrower
 - May or may not allow for recourse
 - Preferred equity
 - Like debt with participation
 - Presales
 - Leasebacks
 - REITs

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Mortgages

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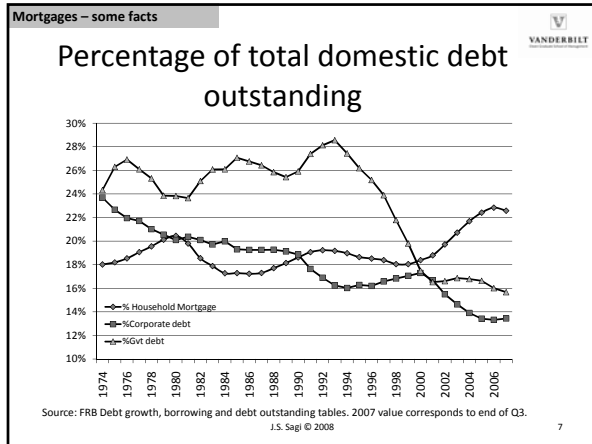

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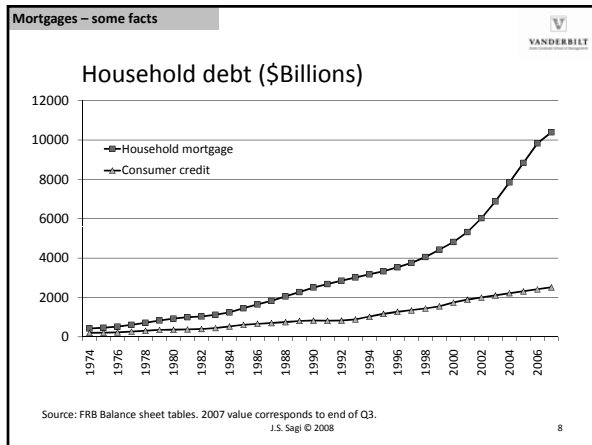
Mortgages - history

- Contracts date to the time of ancient Babylon.
- Example of a mortgage on real estate in security for a loan. The interest was at the rate of eleven and one-third per cent (fourteenth year of Nabopolassar, 611 BCE):
 - ONE mana of money, a sum belonging to Iqisha-Marduk, son of Kalab-Sin, (is loaned) unto Nabu-etir, son of _____, son of _____. Yearly the amount of the mana shall increase its sum by seven shekels of money. His field near the gate of Bel is Iqisha-Marduk's pledge. (This document bears the name of four witnesses, and is dated) at Babylon, Tammuz twenty-seventh, in the fourteenth year of Nabopolassar, (the father of Nebuchadnezzar).

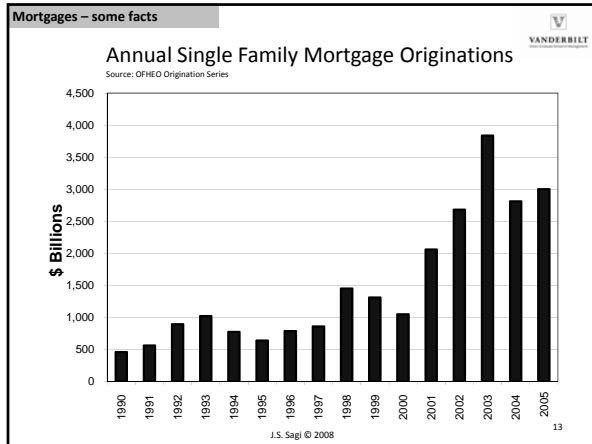
Source: <http://www.fordham.edu/halsall/ancient/mesopotamia-contracts.html>

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Types of mortgages

Types of mortgages

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Types of mortgages

Mortgage Contracts: 5 Key Features

- These affect the value and risk of the contract through time
 1. Maturity and payment frequency
 2. Interest rate charged on remaining principal each period
 3. Amortization schedule
 4. Borrower call options (for prepayment or default)
 5. Recourse to borrower assets (in addition to real estate collateral)

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Types of mortgages




Five key features: 1

- **Maturity**
 - Most popular
 - In the US: 30 years
 - Different in other countries:
 - Canada: 25 years
 - UK: 25-30 years
 - Iceland: 40 years
- **Payment frequency**
 - Typically monthly

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Types of mortgages




Five key features: 2

- **Interest rate charged on remaining principal each period**
 - Fixed
 - Variable
 - Adjustable Rate Mortgages
 - Initial rate (teaser)
 - Adjustment period
 - Benchmark rate + spread
 - Caps
 - Hybrid
 - X-in-Y mortgages
 - X is the number of years of fixed rate applied to the principal
 - Y is the adjustment period used when the fixed-rate period ends.

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Types of mortgages



Five key features: 3

- **Amortization schedule**
 - Determined by how much payment is made each period in excess of the interest due on the principal
 - Level payment amortization
 - As principal is paid, less interest is charge and more of the level payment is applied to the principal (until it is all paid off).
 - Most common
 - Balloon mortgages:
 - E.g., "30 due-in 5"
 - Fixed payments are made as if the loan is fixed rate + fixed payment amortized over 30 years, but after 5 years the entire remaining principal must be paid (or, more likely, refinanced)
 - Growing equity mortgage
 - Starts like level payment.
 - Paid off faster
 - Graduated payment mortgage
 - Starts below equivalent level payment mortgage
 - Payment is subsequently increased
 - May involve a period of negative amortization
 - Custom

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Types of mortgages

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Five key features: 4

- Most common borrower's options
 - Default
 - Skip payments
 - Usually involves interest penalties
 - Foreclosure
 - If payments are in default for long enough...
 - Very bad for the lender if the borrower is 'upside down' and the loan is non-recourse
 - Prepayment
 - Sale of property
 - Refinancing

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Types of mortgages

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Five key features: 5

- The presence of recourse affects the likelihood and payoffs of default
 - Felix Salmon (a blogger) on recourse
 - "... no one is going to have a real handle on mortgage losses unless and until someone manages to get a handle on the percentage of mortgage loans which are non-recourse. If your house falls in value and you have a non-recourse mortgage, then it makes perfect economic sense for someone in a negative-equity situation to simply walk away – something known as "jingle mail". But given the amount of refinancing going on during the last few years of the mortgage boom, I suspect that the vast majority of mortgages are not non-recourse. (Refis are never non-recourse.)"

<http://seekingalpha.com/article/58518-are-subprime-losses-being-exaggerated>

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Valuation of mortgage securities

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Valuation of mortgage securities

- In order to understand the capital markets for RE, we need to understand how the market prices the basic RE financing instrument
 - Mortgages are complicated assets
 - Bond-like attributes
 - Interest rate sensitivity
 - Built in timing options
 - Idiosyncratic risk exposure
 - Sensitivity to housing market
 - Illiquidity
 - If mortgages were easy to understand, the words 'subprime' and 'meltdown' wouldn't belong together....

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Valuation of mortgage securities

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Our approach

- Start simple
 - No options
 - No default
 - Take account of features 1-3
- Progressively add complications
 - Prepayments & default
 - Feature 4
 - Won't spend time on recourse loans
 - Although it is important....

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Valuation of mortgage securities

Simple mortgage valuation

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Without the option features....

- The mortgage is a risk-free fixed income security
 - Value is determined by prevailing loan rates for treasury securities

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Valuation of mortgage securities

Simple mortgage valuation

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Let's focus on the 'boxes' first

- Specify the payment schedule of the mortgage
- Use prevailing treasury rates to calculate PV
- Reverse if necessary
 - I.e., Given an amount to be borrowed, what could the payment schedule look like?

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Valuation of mortgage securities Simple mortgage valuation

The loan rate of the level payment mortgage

- To borrow \$100,000 need 360 equal monthly payments \$489.48
- The mortgage rate is the yield to maturity of mortgage
 - i.e., the YTM (APR compounded monthly) of a simple annuity
- Recall:

$$PV = \text{payment} \times \frac{1}{\text{ym}/12} \left(1 - \frac{1}{\left(1 + \frac{\text{ym}}{12}\right)^{\# \text{payments}}} \right)$$
- Plugging in PV = \$100,000, payment = \$489.48, #pmnts = 360
 - ym = 0.04208
- Or, use Excel's rate function
 - "=12*RATE(360, 489.48, -100000)"

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Quotes from Bankrate.com 2/26/2008

- How do they get the numbers?

Date	"APR"*	Points	Rate (YTM)	Fees	Quoted monthly pmt	Calculated monthly pmt	PV (with fees)	Calculated "APR"
Aasent	6.29	0	6.25	\$690	\$1,016	\$1,015.93	\$ 164,310	6.29
E-Loan	6.899	0	6.875	\$410	\$1,084	\$1,083.93	\$ 164,590	6.90
AimLoan	6.254	0	6.125	\$2,245	\$1,003	\$1,002.56	\$ 162,755	6.25
Quicken Loans	6.644	0	6.5	\$2,443	\$1,043	\$1,042.91	\$ 162,557	6.64
BoA	6.819	0	6.75	\$1,155	\$1,070	\$1,070.19	\$ 163,845	6.82

Amount borrowed \$ 165k
 *PV with fees = Amount borrowed less fees paid (fees could include points)

- Why do the APR*s differ so much across lenders?

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Why the spread between our ytm and Bankrate quotes?

- Almost 2.7% !!!
- In PV-terms, what does it mean?
 - PV of a risk-free loan (without options) paying \$1,084 is
 - \$204,297.52 × (\$1,084 / \$1000) = \$221,458.52
 - E-Loan will give you a total funds amounting to \$164,590
 - Is E-Loan making a positive NPV of \$56,869 on this loan??
 - If not, what does this amount really represent?

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Let's move on to other types of mortgages

- Interest only:
 - Borrow \$100,000 (risk-free, no options)
 - Constant (interest-only) payments for 10 years and then payments increase to a constant level so as to fully amortize the loan.
 - This means that in year 10, the value of the security is \$100k

Mortgage payments

PV of these cash flows = PV \$100k due in 10 years

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Interest-only mortgage example

Year 10, owe	\$100,000
PV(0) of year 10 balloon payment	\$66,731
PV(0) of \$1 monthly in years 10-30	\$101.38
PMT in years 10-30	\$658.22

PV(0) of interest payments	\$33,269
PV(0) of \$1 monthly in years 0-10	102.92
PMT in years 0-10	\$323.26

PV(0) of all payments \$100,000.00

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Interest-only mortgage example


- What is the ytm that would be quoted for this loan?
 - It's an 'interest-only' loan, meaning that the monthly payment should correspond to the quoted ytm interest in years 0-10
 - $\$323.26 = \$100k \times (\text{ytm}/12)$
 - ytm = 3.88%
 - Comparing that to Bankrate.com rates
 - Quicken Loans: 7.00%
 - AimLoan: 6.375%
 - Spread is even bigger than with the FRMs!
 - Why?

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Let's look at an adjustable rate mortgage (ARM)

- The silliest ARM you've ever seen:
 - Risk free, no options
 - Initial rate: 1-month treasury
 - Adjustment period: monthly
 - So benchmark rate is the 1-month treasury rate, and the spread is zero.
 - Caps
 - None
- How are the payments determined?



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The silly ARM

- Let $r_{t,t+1}$ represent the actual (not expected) 1-month treasury rate between date t and t+1
 - Because interest rates fluctuate, nobody really knows $r_{t,t+1}$ until date t.
- Let P_t represent the remaining principal at date t
- The next payment (i.e., at date t+1) is determined as if the loan is an annuity with ytm equal to $r_{t,t+1}$:

$$P_t = \frac{C_{t+1}}{r_{t,t+1}/12} \left(1 - \frac{1}{\left(1 + r_{t,t+1}/12 \right)^{n_{\text{pmts left}}}} \right)$$
 We know: P_t and $r_{t,t+1}$. Solve for C_{t+1}
- Will the series of payments (C_{t+1} , C_{t+2} , C_{t+3} , ...) have a PV equal to P_t ?

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The silly ARM

- Principal at date t+1:
 - $P_{t+1} = P_t (1 + r_{t,t+1}/12) - C_{t+1}$
- PV of payments:
 - $PV(C_{t+1}, C_{t+2}, C_{t+3}, \dots) = PV(C_{t+1} \text{ \& } P_{t+1})$

$$= \frac{C_{t+1}}{1 + r_{t,t+1}/12} + \frac{P_{t+1}}{1 + r_{t,t+1}/12} = \frac{C_{t+1} + P_{t+1}}{1 + r_{t,t+1}/12}$$

$$= \frac{C_{t+1} + P_t \left(1 + r_{t,t+1}/12 \right) - C_{t+1}}{1 + r_{t,t+1}/12} = P_t \quad \text{YES!}$$

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The silly ARM: what's the lesson?

- If there is no risk, no caps, no option features, and the rates adjust every month, then ARM rates should coincide with the 1-month treasury rates.
- What do ARM rates look like in practice?
 - Citibank: 4.715%
 - Compare with 1-year treasury rate from Bloomberg (interpolated)
 - 2.01%
 - A spread of about 2.7% again

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Wells Fargo ARM Example

<http://www.wellsfargo.com/mortgage/rates/assumptions.jhtml>

Loan and APR Information for Adjustable-Rate Loans (as of 02/27/2008 11:06 AM Eastern)

	Conforming		Jumbo
	5-in-one ARM	10-in-one ARM	5-in-one ARM
Interest Rate	5.25%	7.13%	6.38%
Total Points	1	1	1
APR	5.39%	6.62%	5.75%
Payment Term	30 yrs	30 yrs	30 yrs
Estimated Additional Prepaid Finance Charges	\$2,000	\$2,500	\$2,500
Loan Amount	\$145,000	\$450,000	\$450,000
Down Payment	20.00%	20.00%	20.00%
Initial Monthly Principal & Interest Payment	\$800.70	\$3,031.74	\$2,807.42
Time Initial Rate is Fixed	5 yrs	10 yrs	5 yrs
Adjustment after Initial Period Expires	Annual	Annual	Annual
Maximum Lifetime Caps Over/Under Original Rate	5%	5%	5%
First Adjusted Payment	\$790.87	\$2,582.72	\$2,489.74

Jumbo loans are those exceeding \$417k. Conforming loans are below \$417k, and require a down payment of at least 20% and a good credit on the part of the borrower. For conforming and jumbo adjustable-rate mortgages (ARMs), at adjustment your new mortgage rate will be the average of the Interbank offered rates for one-year U.S. dollar-denominated deposits in the London market (LIBOR) as published in the *Wall Street Journal*, plus a margin of 2.25% subject to annual and lifetime adjustment caps.

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Computing APR on ARMs (5-Year ARM Example)

5-Year ARM	After 5 Years	For APR, set cash flow, compute YTM	
Rate = 5.25%	Libor = 3.16%	Date 0	-\$145,550
Term = 30 Years	Spread = 2.25%	Dates 1 to 60	\$800.70
Loan = \$145,000	Rate = 5.41%	Dates 61 to 360	\$822.69

Initial Payments = $\$800.70 = PMT(0.0525/12, 360, -145000, 0)$

Balance after 5 yrs: $\$147,024.34 = PV(0.0525/12, 300, -800.70, 0)$

Payment after 5 yrs: $\$822.69 = PMT(0.05411/12, 300, -147024.34, 0)$

YTM $5.39\% = PV(1/12, 60, -800.70) + PV(1/12, 300, -822.69) / POWER(1 + 1/12, 60) - \$145,550$

If the initial ARM rate = initial index plus margin, then APR is computed just as with FRM. If ARM initial interest rate differs from fully indexed rate then the convention is to compute the payments by changing the interest rate as quickly as allowed by the contract until you reach the fully indexed rate. You then calculate the APR based on the multiple payment streams.

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Common Features on Adjustable Rate Mortgages

- Mortgage payment resets with movements of a pre-determined index:
 - 3-Month T-Bill
 - 1-Year constant Maturity Treasury rate
 - LIBOR
 - COFI (Cost of Funds Index)

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Common Features on Adjustable Rate Mortgages

- Reset periods are usually monthly, or annual
 - including an initial period of fixed rates.
- Life-of-loan caps required by Regulation Z “truth-in-lending” legislation.
- Periodic interest rate and payment caps are common.
 - Floating rate is still charged but the additional required payment is added to the principal
 - Allows for negative amortization.

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ARM Example: Payment CAPS And Negative Amortization

- 70,000 borrowed
- 30 Year maturity
- Adjustments period: 6 months
- Index: 6-month T-Bill rate
- Margin = 1.30%
- Payment change cap = 7.50%
 - Payment cannot increase by more than 7.50% per 6-month interval
- No interest rate change cap
- Lifetime rate level cap: 4.00%
- End of every five years: Payment cap lifted to fully amortize loan.

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ARM Example/1

End of Month	Balance At Date	Index	Change in Index	Index plus margin	Rate after cap	Payment owed next 6 months	Minimum payment required	Periods Left
0	\$70,000	7.20%		8.50%	8.50%	\$538.24	\$538.24	360
6	69,741	9.40%	2.20	10.70%	10.70%	649.92	578.61 (7.5% Cap)	354
12	70,006	9.06%	1.86	10.36%	10.36%	636.34	622.00 (7.5% Cap)	348
18	69,898	11.85%	4.65	13.15	12.50% (4% Cap)	749.77	668.65 (7.5% Cap)	342

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ARM Example/2

- Payment in months 1-6: solve

$$70,000 = \frac{C}{.085/12} \left(1 - \frac{1}{(1 + .085/12)^{360}} \right) \Rightarrow C = \$538.24$$
- Remaining principal after 6th payment:

$$P = \frac{\$538.24}{.085/12} \left(1 - \frac{1}{(1 + .085/12)^{354}} \right) \Rightarrow P = \$69,741.01$$
- Payment in months 7-12: solve

$$\$69,741.01 = \frac{C}{.107/12} \left(1 - \frac{1}{(1 + .107/12)^{354}} \right) \Rightarrow C = \$649.92$$
 - This is an increase of $(649.92 - 538.24)/538.24 = 20.75\%$
 - The cap kicks in and the minimum payment is \$578.61

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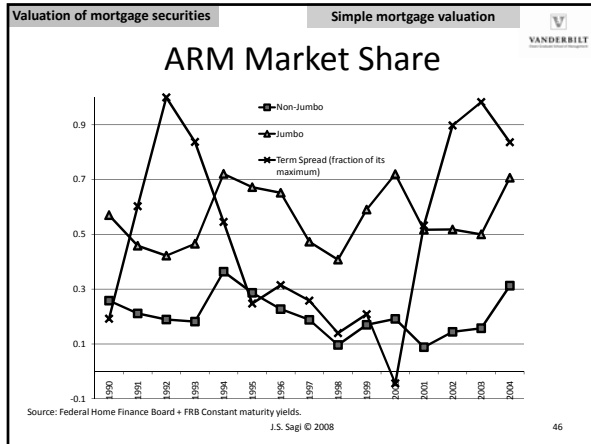
ARM Example/3

- Remaining principal after 12th payment:
 - Have to include gap between owed payment and minimum payment

month	payment owed	payment made	Contribution of shortfall to principal
13	\$649.92	\$ 578.61	\$71.31
14	\$649.92	\$ 578.61	\$143.26
15	\$649.92	\$ 578.61	\$215.84
16	\$649.92	\$ 578.61	\$289.08
17	\$649.92	\$ 578.61	\$362.96
18	\$649.92	\$ 578.61	\$437.51

Principal if full payments are made \$69,568.86 Principal owed after 18th payment \$70,006.37
- Notice that the shortfall in payment leads to negative amortization

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Option-Based ARMs

- These ARMs give borrower option to reduce payment all the way down to a specified minimum.
 - If payment < accrued interest, then negative amortization.
 - Advertised rates as low as a 1%, but this only determines the minimum payment. The accrual rate is much higher.
- The loans have effects on reported lender income:
 - Total accrued interest is counted as income, even if not paid. This assumes negative amortization will be repaid.
 - If loan defaults, then income accruals must be reversed. A big fear is the loans will, indeed, have very default rates.

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From Option ARMs to Predatory Lending

- Predatory Loans refer to loans made by unscrupulous lenders to high-risk borrowers where lenders expect the borrowers to default. Motives:
 - They set extremely high interest rates.
 - They set very high prepayment fees, then induce the borrowers to prepay (by offering cash out).
 - Ultimately, they get the house because the mortgage has ballooned to exceed the house value.
- Cities, such as Oakland, passed Predatory Lending Laws making even MBS investors liable. But no one would lend.
- Might be more sensible is to take direct action against the bad lenders.


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More Alternative Mortgages

- So far we have considered IOM, FRM, and ARMs.
 - infinite set of choices for payment patterns.
 - Depends on the needs of lenders and borrowers:
- More animals in the zoo:
 - Graduate Payment Mortgages (GPMs)
 - start with low payments, then graduate to higher.
 - Price Level Adjusted Mortgages (PLAMs)
 - keep payments constant in real terms.
 - Reverse Annuity Mortgages (RAMs)
 - allow older borrowers to tap home equity to provide for living expenses.
 - Shared Appreciation Mortgages (SAMs)
 - allow a 3rd party to pay part of the down payment, then participate in price gains.


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Valuation of mortgage securities Accounting for options 

Incorporating the option features

- Generally, use of the options embedded in a mortgage is subject to the discretion of the borrower
 - The contract is a 'zero-sum game' between the lender and borrower
 - Options give the borrower an advantage relative to a contract without options
 - This makes the mortgage less valuable to the lender
 - Explains why quoted mortgage rates are higher than what we calculated using the simple approach
 - E.g., Spread of 2.7% the FRM we saw earlier


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Valuation of mortgage securities Accounting for options 

Incorporating the option features

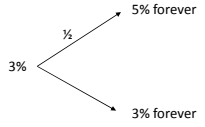
- A comprehensive treatment is beyond the scope of this course
 - Requires sophisticated financial engineering
- Our approach:
 - Attempt to elicit some valuable intuition from a few stylized examples
 - Describe qualitatively what is done in practice

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Valuation of mortgage securities Accounting for options 

Stylized example of the prepayment option

- Borrow \$1M, promise to pay back a level perpetuity
- 1-yr T-Bill rate is 3%
- Rates are subsequently expected to rise to 5% and stay there forever, or decline to 1% and stay there forever
 - Risk-neutral probability of each scenario is 1/2




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Valuation of mortgage securities Accounting for options 

Stylized example of the prepayment option: YTM without prepayment option

- If the coupon is C, in one year the value of securities held + cash flow is:
 - $C + C/0.05 = 21C$ (if rates rise to 5%)
 - $C + C/0.01 = 101C$ (if rates fall to 1%)
- The present value of this should be set to \$1M
 - $PV = \$1M = \frac{1}{2}(21C + 101C)/(1+3\%) = 59.22... C$
 - So $C = \$1M/59.22... = \$16,885.25$ & $ytm = C/\$1M = 1.689\%$
 - In the low interest state, the obligation would be worth $C/0.01 = 100C = \$1.69M$, servicing a principal of only \$1M.
 - It would be good to refinance!


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Valuation of mortgage securities Accounting for options 

Stylized example of the prepayment option: YTM with a prepayment option

- Refinancing in the low-rate state means paying \$1M to settle the debt:
 - $C + C/0.05 = 21C$ (if rates rise to 5%)
 - $C + \$1M$ (if rates fall to 1%)
- The present value of this should be set to \$1M
 - $\$1M = \frac{1}{2}(21C + C + \$1M)/(1+3\%) = 10.67... C + \$0.485...M$
 - So $C = \$48,181.82$ and $ytm = 4.818\%$
 - A spread of 3.130%
- The spread is large because the lender gives up a lot through the prepayment option
 - about \$690k in the low interest state
 - This requires compensation in the form of a higher spread

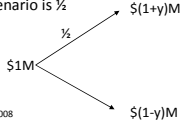
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Valuation of mortgage securities Accounting for options 


Stylized example of the default option

- Purchase an asset for \$1M, using a loan of \$800k (a Loan-to-value, or LTV, of 80%).
 - All obligations will be settled in one year.
- 1-yr T-Bill rate is 5% and expected to remain at 5% forever
- Asset value can increase by $y\%$ or decrease by $y\%$
 - Risk-neutral probability of each scenario is $\frac{1}{2}$

Caveat: This scenario tree is for the asset and not for interest rates (as was the case with the previous example).




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Stylized example of the default option: YTM without the default option

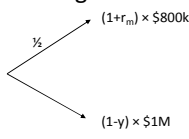
- Must pay back $\$800k \times (1+5\%) = \$840k$ or hand over the asset to the bank
 - If asset increases by y , default is suboptimal
 - It's not a good idea to give something worth $(1+y) \times \$1M$ to satisfy an obligation of \$840k.
 - If asset decreases by y , default is optimal if
 - $\$840 > (1-y) \times \$1M$,
 - i.e., $y > 0.16$
 - If $y < 0.16$, then the bank will get its money and the debt is risk-free
 - $y_{tm} = 5\%$
 - What is the y_{tm} in the case that $y < 0.16$?

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Valuation of mortgage securities Accounting for options 

Stylized example of the default option: YTM with the default option

- If $y > 0.16$, the bank faces the following scenario tree:
 - r_m is the mortgage rate
 - i.e., the y_{tm}
 - The PV of this must equal \$800k
 - $\$800k = \frac{1}{2} ((1+r_m) \times \$800k + (1-y) \times \$1M) / (1+5\%)$
 - Assumes $y > 0.16$
 - Solving for r_m gives: $r_m = 1.25y - 0.15$
 - Plotting this....



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Stylized example of the default option:
YTM with the default option

Notice that there is very little sensitivity to small price shocks. The effect is then ratcheted beyond the threshold of $y = 0.16$. If RE price volatility is 20% or more, the yield spread on quoted mortgages could be larger than 5%.

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Stylized example of the default option:
YTM with the default option

- For more general LTV and (constant) T-Bill rate assumptions, this (Mickey Mouse) model implies
 - No default for y below: $1 - LTV \times (1+r)$
 - Low LTV creates a buffer against default
 - The lower the LTV, the better
 - Above threshold:
 - $r_m = y/LTV - (1/LTV - 1 - 2r)$
 - Slope is greater than 1 if $LTV < 1$
 - Ratcheting effect


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Let's look at a 'subprime loan'.... (LTV = 0.95, $r = 5\%$)

Even a little bit of price volatility implies non-trivial spreads.


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How are mortgages valued in practice?

- Choose a term structure model
 - Complete with risk-neutral probability estimation and fit to the yield curve
 - Need to account for the possibility of changing rates
 - Recall refinancing stylized example
- Choose a model of RE price movement
- Call up your favorite quant-geeks....

- Spreadsheet example....

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Mortgage options: a summary

- Mortgage options reduce the mortgage value
 - Increase the quoted mortgage rate
- Prepayment option
 - More valuable when interest rate volatility is high
 - Value is reduced by presence of prepayment penalties
- Default option
 - Sensitive to interest rate volatility
 - Non-linear, and highly dependent on the LTV ratio

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