Journal of Financial Economics 22 (1988) 103–122. North-Holland

THE BEHAVIOR OF THE VOLATILITY IMPLICIT IN THE PRICES OF STOCK INDEX OPTIONS*

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Received November 1987, final version received June 1988

We examine stock-market volatility around the quarterly expirations of stock index futures contracts and nonquarterly expirations of stock index options, using estimates of the volatility implicit in the option prices. The option prices reflect increases in the volatility of the underlying stock indexes around both quarterly and nonquarterly expiration dates. Analysis of the residual returns on index options provides evidence consistent with an unexpected increase in market volatility around expiration dates.

1. Introduction

The effect of trading in stock index futures contracts on market volatility has been much discussed recently. Particular attention has been focused on the market's volatility at the close of trading on days when futures contracts expire. Unusually large trading volume and price movements at the close of the market on quarterly expiration days have been attributed to the unwinding of arbitrage positions involving futures contracts and the underlying stocks. Stoll and Whaley (1987b) report that the volume of trading in New York Stock Exchange (NYSE)-listed stocks during the last hour on Fridays when index futures contracts expire is nearly double that on other Fridays. They show that the standard deviations of the rates of return on the Standard and Poor's 500 and 100 Indexes during the last hour of trading are significantly greater on days when index futures contracts expire than on other days. Stoll and Whaley also show that the Standard and Poor's 100 Index is significantly more volatile during the last hour of trading on Fridays when only options on the stock indexes expire.

This study examines the behavior of the market volatility implicit in the prices of call options on stock indexes around both the quarterly expirations

*This research was supported through the Financial Markets Research Center at Vanderbilt University by a grant from the Chicago Board of Trade, the Chicago Board Options Exchange, and the Chicago Mercantile Exchange, and by the Dean's Fund for Faculty Research at the Owen Graduate School of Management.

0304-405X/88/$3.50

of stock index futures and the (nonquarterly) expirations of index options. Call options on the Standard and Poor's 500 and 100 Indexes, the Major Market Index, and the New York Stock Exchange Composite Index trade, respectively, on the Chicago Board of Trade, the American Stock Exchange, and the New York Stock Exchange. Observed prices of call options on stock indexes are equated to the dividend-adjusted version of the Black–Scholes option pricing model to infer the instantaneous variances of the rates of return on stock indexes. The time series of these implied volatilities are then examined in days surrounding the quarterly expirations of stock index futures and expirations of options on indexes and individual stocks only. Implied market volatilities provide *ex ante* estimates of market volatility around expiration days. Since implied volatilities closely reflect investors' expectations about the potential range for future price movements, they are of particular interest in studying expiration-day effects.

Latane and Rendleman (1976) are the first to examine the empirical properties of implied volatilities. They show that the volatilities implied by call option prices on individual stocks predict future volatility better than predictors based on historical stock price data. Chiras and Manaster (1978) reach a similar conclusion using Merton's (1973) constant proportional dividend yield model. Patell and Wolfson (1979) examine changes in implied volatilities in days surrounding annual earnings announcements. They show that for an option with a given expiration date, the implied volatility increases as the earnings announcement date approaches and then falls immediately after the announcement. Patell and Wolfson argue that, since the instantaneous variance of an option can be interpreted as the average variance per unit of time from the valuation date to the expiration date, this behavior can be attributed to greater than average uncertainty at the announcement date due to the information content of the corporate earnings announcement.

Generally, estimates of implied volatility contain noise. Two significant sources of noise are the inability to determine whether option and stock prices reflect bid or ask levels and the failure to observe the option price and the price of the underlying security simultaneously. As a result, option prices with different expiration dates and/or exercise prices will provide different estimates of implied volatilities. Thus an estimate of the implied volatility for a given security must incorporate information from estimates of the volatility implicit in the prices of the different options.

Several approaches have been used to construct weighted-average volatility estimates from volatility estimates of individual options.[1] Whaley (1982) determines the implied volatility using a nonlinear regression procedure that

[1]For example, Schmalensee and Trippi (1978) and Patell and Wolfson (1979) use an arithmetic average, Latane and Rendleman (1976) use an average weighted by the partial derivative of the option with respect to the volatility, and Chiras and Manaster (1978) use an average weighted by the elasticity of the option to volatility.

minimizes the sum of the squared residual deviations of the model prices for the sample options around their observed prices. Park and Sears (1985), who employ the Black (1976) pricing model for options on futures contracts, use Whaley's technique to examine the volatility of the underlying futures over the life of the contract for futures contracts on the Standard and Poor's 500 Index and the New York Stock Exchange Composite Index.

We estimate the implied volatilities for each index option expiration series using a generalized-least-squares (GLS) procedure in which actively traded options are weighted more heavily than thinly traded options. The differences between the implied volatilities for expiring options and longer options are then used to study *ex ante* market volatility surrounding both quarterly and nonquarterly expiration days. Section 2 discusses our data and approach. The procedures used to estimate implied volatilities are discussed in section 3. Section 4 describes our empirical tests of the significance of changes in market volatility around expiration days, and the results are presented in section 5. The behavior of the residual returns on options around expiration dates is examined in section 6. We conclude with a brief summary.

2. Data and method

2.1. Model specification

Black and Scholes (1973) derive the value of a European call option on a non-dividend-paying-stock as a function of the current stock price, the time to expiration, the exercise price, the risk-free rate of interest, and the volatility of the underlying stock. Merton (1973) shows that the Black–Scholes formula can be used to value American call options on non-dividend-paying-stocks because early exercise is never optimal. The formula also can be used to value an American call option on a stock that pays a finite number of known dividends over the life of the option as long as early exercise is not optimal. In this case, the call is valued by replacing the current stock price in the Black–Scholes formula with the current stock price minus the present value of the dividends.

The assumption of no early exercise is appropriate for call options written on a broad-based stock index. Early exercise, if it were to occur, would take place just before an ex-dividend instant. At that time, the call option holder will not exercise if the exercisable proceeds of the call, $I_t - K$, are below the lower bound of the unexercised call after the index has gone ex-dividend, $I_t - D_t - Ke^{-r\tau}$. Thus, early exercise will not occur at ex-dividend instant t if

$$D_t \leq K(1 - e^{-r\tau}), \tag{1}$$

where

D_t = dividend paid at time t,
r = riskless rate of interest,
K = strike price,
τ = time to expiration.

This condition is unlikely to be violated by any of the cash dividends paid on the index during the option's life.

The dividend-adjusted version of the Black–Scholes formula,

$$C(K,\tau) = (I - PV(D(\tau)))N(d_1) - K\exp(-r\tau)N(d_2), \qquad (2)$$

where

I = current level of the index,
$PV(D(\tau))$ = present value of all dividends paid during the option's time remaining until expiration,
$N(\cdot)$ = cumulative normal density function,
d_1 = $\{\ln[(I - PV(D(\tau)))/K\exp(-r\tau)] + \frac{1}{2}\sigma^2\tau\}/\sigma\sqrt{\tau}$,
d_2 = $d_1 - \sigma\sqrt{\tau}$,

is the appropriate model for the valuation of call options on stock indexes.[2] Taking the prices of index call options as given, eq. (2) can be used to estimate the market volatility implicit in these prices. The remaining parameters of the option pricing model – the current index value, the present value of future dividends, the riskless rate of return, and the contractual features of the option itself – are known or can be estimated accurately.

2.2. Data

The data consist of daily closing prices, contract volumes and the closing level of the underlying index for all call options on the Major Market Index, the New York Stock Exchange Composite Index and the Standard and Poor's 100 Index from the start of trading in 1983 through December 31, 1986.[3,4]

[2] Eqs. (1) and (2) implicitly assume that the stock index drops by the amount of the cash dividend on the ex-dividend date. Barone-Adesi and Whaley (1986) provide empirical evidence to support this assumption.

[3] The first available quotation date for Standard and Poor's 100 options is March 11, 1983. Price and volume quotations for options on the Major Market Index, the S&P 500, and the NYSE Composite, respectively, begin on April 29, July 1, and September 23, 1983. The implied volatilities for options on the S&P 500 are not examined in this study because trading in these options was not sufficiently active to generate a continuous time series of implied volatilities.

[4] These data were obtained from the Chicago Board Options Exchange and from Commodity Systems Incorporated of Boca Raton, Florida.

Option quotations with daily volume of less than 100 contracts are eliminated from the sample to insure that the closing price for the option is roughly concurrent with the closing level of the index. Option quotations with closing prices less than $0.25 are eliminated because of the size of the bid–ask spread in relation to the price of the option.[5] Finally, we eliminate quotations violating the lower boundary conditions for American options.

Bid and ask discounts for U.S. Treasury bills are collected daily from the *Wall Street Journal*. The risk-free rate of interest for each option expiration series is computed for each day using the average of the bid and ask discounts for the U.S. Treasury bill whose maturity is closest to the expiration date. The actual dividends paid over the life of each option contract are used as a proxy for expected dividends. The present value of these dividends is computed using the yield on the U.S. Treasury bill maturing closest to the expiration date of each option.

The nature of option price data suggests that some quotations should be more useful than others because they reflect the *ex ante* volatility of the underlying index more accurately. First, since closing prices represent the option price for the last trade of the day, the closing prices for thinly traded contracts are more likely to represent transactions that occur before the close of trading.[6] This implies that the index level reflected in the closing prices of thinly traded options is less likely to be equal to the actual closing level for that day. In this case, an estimate of the implied volatility is biased upward whenever the index level at the close is less than the level at the time of the last trade in the option and vice versa. The final trade of the day for an actively traded contract is likely to occur at or near the close of the market. Consequently, the index level reflected in the reported closing prices of actively traded options will be a relatively accurate approximation of the actual closing level.[7] Table 1 summarizes the distribution of trading volume according to the extent to which an option is in the money for each of four categories of time to expiration. Because trading volume tends to be concentrated in the options that are at the money and just in the money, any lack of synchronization between the closing index level and the closing option price will be minimized for these options.

[5]Phillips and Smith (1980) find that the average percentage bid–ask spread for call options is 30%. However, the average percentage spread for call options priced at more than $0.50 is only 4.5%.

[6]Since the New York Stock Exchange closes at 3:00 p.m. Central Standard Time, whereas trading in index options on the Chicago Board of Trade continues until 3:15 p.m., the reported closing price for any index option contract for which the final trade of the day occurs after 3:00 p.m. should reflect the actual closing level of the index.

[7]Since not all stocks trade continuously, the closing level of the index itself may be a stale quotation. This problem should be of little importance for indexes composed of a relatively small number of actively traded stocks, such as the Major Market Index and the Standard and Poor's 100.

Table 1

Distribution of trading volume for call options on the Standard and Poor's 100 Index, the Major Market Index, and the New York Stock Exchange Composite Index.

Each column contains the proportion of trading volume represented by trading in contracts that are within \$2.50 of being \$$X$ in the money or $-$\$$X$ out of the money for each of four categories of time to expiration and for each of the three indexes included in the sample. The sample includes all contracts traded between March 11, 1983 and December 31, 1986.

	\$$X$ out of the money					\$$X$ in the money				
Days to expiration (τ)	−20	−15	−10	−5	0	5	10	15	20	25
$\tau < 30$										
S&P 100	0.001	0.004	0.023	0.158	0.494	0.254	0.059	0.007	0	0
Major Market	0.001	0.003	0.013	0.110	0.468	0.254	0.099	0.034	0.013	0.004
NYSE	0.003	0.006	0.023	0.088	0.692	0.163	0.017	0.006	0.001	0
$30 < \tau < 60$										
S&P 100	0.001	0.005	0.021	0.104	0.285	0.308	0.187	0.070	0.016	0.004
Major Market	0.001	0.003	0.014	0.075	0.359	0.269	0.143	0.074	0.031	0.030
NYSE	0	0.001	0.011	0.051	0.553	0.299	0.072	0.014	0	0
$60 < \tau < 90$										
S&P 100	0.002	0.009	0.022	0.083	0.218	0.264	0.225	0.124	0.042	0.011
Major Market	0.002	0.006	0.013	0.038	0.627	0.208	0.075	0.024	0.003	0.002
NYSE	0	0	0	0.023	0.734	0.209	0.032	0.002	0	0
$90 < \tau < 120$										
S&P 100	0	0.004	0.019	0.067	0.400	0.269	0.165	0.055	0.016	0.005
Major Market	0	0	0	0	0.870	0.130	0	0	0	0
NYSE	0	0	0	0	0.915	0.085	0	0	0	0

A second limitation of the data is we have no way of knowing whether the last transaction of the day is made at the bid or the ask price. The percentage bid–ask spread tends to be greater for options that are far out of the money. As a result, estimates of the implied volatility for options that are far out of the money will have a significant upward or downward bias depending on whether the close represents a transaction at the bid or the ask price.

3. Estimation of implied volatility

Most prior studies have used call options to estimate implied volatility by estimating the implied volatilities from individual call options numerically and then aggregating these estimates to form a weighted-average estimate. Any investigation of implied volatilities should consider the interactions between a particular weighting scheme and the potential empirical problems noted above. The Black–Scholes option model can be used to examine the weights that different estimation approaches place on individual options in computing an estimate of volatility. For example, Chiras and Manaster (1978) weight the individual options on the basis of the elasticity of the call price to the standard deviation of the stock return. Another approach, which was originally used by Latane and Rendleman (1976), weights the individual options on the basis of the partial derivative of the call price with respect to the standard deviation of the stock return. To illustrate the weighting implied by these approaches, assume an index level of 110, a standard deviation of 15%, a time to expiration of three months, and a riskless borrowing rate of 8%. Fig. 1 shows that weighting on elasticity tends to exacerbate any potential bias created by the bid–ask spread because it places the greatest weight on the out-of-the-money options. Fig. 1 also shows that weighting on the partial derivatives of the call price with respect to the standard deviation of the stock return tends to give the greatest weight to the options that are at the money.

We estimate implied volatilities using a GLS version of Whaley's (1982) nonlinear regression procedure. With this approach, each observation is weighted in proportion to the day's total trading volume as a percentage of the total trading volume of its expiration series. Table 1 shows that this weighting scheme effectively places the greatest weight on those options that are either at the money or one strike price (five dollars) out of the money.[8] Consequently, it gives the greatest weight to the option prices that are most sensitive to the volatility of the underlying stock index. In addition, since thinly traded options and far-out-of-the-money options receive relatively little weight, this scheme tends to minimize the extent to which the estimates of implied

[8]The OLS procedure followed by Whaley implicitly weights the at-the-money option most heavily. Since the prices of in-the-money and out-of-the-money options are relatively insensitive to the volatility of the underlying asset, these options have relatively less impact on the regression coefficient.

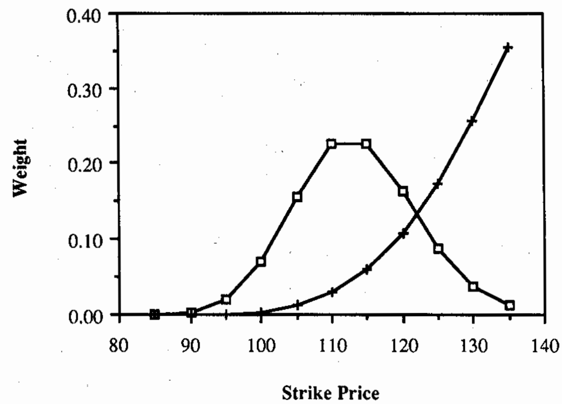

Fig. 1. Comparison of the methods used to weight the estimates of volatility from individual options.

The weights from the price elasticity method (labeled +) and the partial derivative method (labeled □) are shown for an index level of 110, an instantaneous standard deviation of 15%, a time to expiration of three months, and a riskless borrowing rate of 8%.

volatility are affected by noise caused by either nonsynchronous trading or the size of the bid–ask spread.

Let $C_k(\sigma_0(\tau))$ denote the theoretical price of an option having a strike price indexed by k and expiration at τ, given an estimate $\sigma_0(\tau)$ of the volatility of the return on the stock over the time to expiration. The actual price of option k with expiration at τ is denoted by $C_{k\tau}$. Define

$$Y_k = C_{k\tau} - C_k\big(\sigma_0(\tau)\big), \tag{3}$$

and let N denote the number of strike prices with expiration at τ. Given an initial estimate of volatility implicit in the prices of options with expiration at τ of $\sigma_0(\tau)$, a new estimate of the implied volatility, $\sigma(\tau)$, is chosen so as to minimize

$$\sum_{k=1}^{N} (\delta_k Y_k)^2, \tag{4}$$

where δ_k is the proportion of trading volume in options expiring at τ represented by trading in contract k. At each iteration of this process, the new estimate of the implied volatility is given by

$$\sigma(\tau) = \sigma_0(\tau) + [(\Omega X)'(\Omega X)]^{-1}(\Omega X)'\Omega Y, \tag{5}$$

where

Ω = an $N \times N$ diagonal weighting matrix whose diagonal elements equal the percentage of the trading volume in a given expiration series represented by trading in that contract,

X = an $N \times 1$ vector whose elements are the partial derivatives of the call options expiring at τ with respect to the underlying volatility, evaluated at $\sigma_0(\tau)$,

Y = an $N \times 1$ vector whose elements are defined by eq. (3),

and the prime is used to signify the transpose of a matrix.

If the estimate converges to within a given tolerance level

$$|\sigma(\tau) - \sigma_0(\tau)| < \alpha, \tag{6}$$

then the estimate $\sigma(\tau)$ is taken as acceptable. If the estimate is not within the desired tolerance, the procedure is repeated using $\sigma(\tau)$ in place of $\sigma_0(\tau)$. This approach is implemented each day for every option in the sample except for days when the contracts closest to expiration are due to expire in one day or less. On these dates, the short-term options are excluded from the sample because the price of the option in relation to its exercisable value is low enough to make the bid–ask problem an issue.

4. The behavior of implied volatilities around expiration dates

We use the time series of daily estimates of implied volatilities to examine market volatility around the monthly expirations of stock index options. Each of these monthly expiration days can be thought of as an event as commonly defined in the event-study literature. The sample includes two types of events. The first is the quarterly expiration of stock index futures, or the so-called 'triple witching hour' at which the index options, index futures, and the individual stock options expire simultaneously. The second event type is the nonquarterly expiration of the index options.[9] We stratify the sample in this manner to assess whether the unusual activity at the 'triple witching hour' documented by Stoll and Whaley (1987a, b) is endemic to all expiration dates or is restricted to dates on which index futures contracts expire.

The volatility implicit in the price of a call option on a stock market index represents an *ex ante* forecast of the volatility of that market index over the life of the option. Although the Black–Scholes option pricing model assumes that the instantaneous variance in returns on the underlying security is constant over the option's life, Merton (1973) shows that the Black–Scholes formula is also appropriate when the instantaneous variance changes over the

[9] Index options have monthly rather than quarterly expiration cycles.

life of the option as a known function of time. In this case, the instantaneous variance of return that should be used in the Black–Scholes formula can be interpreted as the average of the instantaneous variance of the underlying security at each moment over the life of the option. The average variance rate over the life of an option that expires at τ is defined by

$$\sigma^2(\tau) = \tau^{-1} \int_0^{\tau} \sigma^2(t)\,\mathrm{d}t. \tag{7}$$

The notion that the volatility implicit in the price of a call option is an average of the volatility of the underlying security at different points over the life of the option is particularly useful in studying changes in market volatility around expiration dates. Under the null hypothesis that the instantaneous variance of the return on the market index underlying an index option is constant over time, the implied volatilities for call options with different times to expiration can be compared to test for systematic differences in volatility around expiration dates.[10] The null hypothesis is

$$\mathrm{H}_0: \quad \sigma(\tau_i) - \sigma(\tau_j) = 0 \quad \text{for all} \quad \tau_i < \tau_j. \tag{8}$$

If market volatility increases temporarily around expiration days, then the implied volatility for the option with the least time to expiration should be significantly greater than the implied volatilities for options with more distant expiration dates, since all outstanding options span the nearest expiration day. For example, assume that the quarterly expiration day is the only day typified by volatility increases. Since volatility is an average of the daily volatilities over the option's life, an option with three days to expiration places much more weight on the expiration-day volatility than an option with 33 days to expiration. Consequently, the implied volatility of the option with three days to expiration will be greater than the implied volatility of the option with 33 days to expiration, all else being equal.

We test the null hypothesis by constructing a time series of the day-by-day differences in implied volatilities in the window of time surrounding expiration days for all possible pairs of options having different times to expiration. On day t, measured in relation to expiration at day 0, the difference between the implied volatilities for options whose expirations occur respectively at τ_i and τ_j ($\tau_i < \tau_j$) is given by

$$\varepsilon_{tij} = \sigma_t(\tau_i) - \sigma_t(\tau_j). \tag{9}$$

[10] The assumption that the returns on the underlying stock index follow a stationary distribution is relatively mild, since the options included in the sample have a maximum of four months to expiration.

Under the null hypothesis, ε_{tij} has an expected value of zero at each date for all i and j. The extent to which *ex ante* market volatility temporarily has increased on a given day in the event window can be measured by the sample averages of these volatility differences for each of the days in the event window. The average difference in the implied volatilities of options with different times to expiration for a given day defined in relation to an expiration date is similar in some respects to the average daily residual return in the window of time surrounding an information event.

Consider an event window of w days before and w days after each expiration day. The average difference in volatilities on day s (in relation to the expiration at day 0) for two options that expire at τ_i and τ_j can be expressed as

$$AVD_{sij} = \frac{1}{E} \sum_{v=1}^{E} \varepsilon_{sij}^{v}, \tag{10}$$

where E represents the number of expiration dates of a given type and the superscript v is used to denote the vth expiration date.

The statistical significance of the average volatility difference for each day s is the t-statistic

$$TAVD_{sij} = \frac{AVD_{sij}}{SE}, \tag{11}$$

where

$$SE = \frac{1}{\sqrt{E}} \left(\frac{1}{M-1} \sum_{s \notin W} \sum_{i=1}^{T-1} \sum_{j=i+1}^{T} \left(\varepsilon_{sij} - \bar{\varepsilon}\right)^2 \right)^{1/2}, \tag{12}$$

with

$$\bar{\varepsilon} = \frac{1}{M} \sum_{s \notin W} \sum_{i=1}^{T-1} \sum_{j=i+1}^{T} \varepsilon_{sij}, \tag{13}$$

where W refers to the set of all days within an interval w days before and w days after a quarterly expiration date, M denotes the total number of observations not contained in the set W, and T denotes the number of distinct expiration dates.

5. Evidence on market volatility

Stoll and Whaley (1987b) argue that the cash-settlement feature of stock index futures contracts contributes to the above-average trading volume and market volatility observed on quarterly expiration days. If order imbalances due to index arbitrage create price pressure, the stock indexes underlying futures contracts will be more volatile on the quarterly expiration days. Moreover, this implies that the volatility implicit in the prices of call options on stock indexes should be significantly higher on and before quarterly expiration dates.

In this section, we examine the extent to which increases in market volatility on quarterly expiration dates are reflected in the prices of call options on stock indexes. We find evidence that the market anticipates higher volatility. In order to determine whether these results are limited to quarterly expirations, we also examine the behavior of implied volatility around nonquarterly expirations. We find that the increase in volatility is smaller than that around quarterly expirations.

Section 4 shows that above-average market volatility before the quarterly expiration of index futures contracts would cause the implied volatilities of expiring call options on the index to be greater than the implied volatilities of call options having longer times to expiration. Table 2 reports the average differences in the implied volatilities (nearest expiration minus more distant expiration) for call options on the S&P 100 Index around the 15 quarterly expiration dates from June 1983 through December 1986. The average differences in implied volatilities are reported for each day in a nine-day window beginning four days before and ending four days after the expiration date. Although estimates of implied volatilities may be biased upward when an index option is optimally exercised just prior to an ex dividend date, a check of the conditions necessary to ensure no early exercise reveal that early exercise is never optimal for options traded within the nine-day window we examine.

The first three rows in table 2 contain the average differences between the implied volatility for the expiring series of options and the implied volatility for each of the three expiration series spanning the quarterly expiration date. In each case, the average difference between the implied volatility for the expiring option and the implied volatility for the option spanning the expiration date is positive and statistically significant.[11] Note that the difference between the implied volatilities of the expiring option and the longer option increases monotonically prior to the expiration day. In addition, for a given day in relation to the expiration date, the difference between the implied

[11]Whaley (1982) finds that the longer the time to expiration, the greater the model price in relation to the actual call price. Since estimates of implied volatilities equate model prices to actual prices, this suggests that estimates of the implied volatilities for options with longer times to expiration may be biased downward. The average differences between the implied volatilities of options expiring sooner or later are positive but statistically insignificant outside the window surrounding the quarterly expiration date. The results presented in tables 2 through 4 would be unchanged had the out-of-sample average difference for each pair of options been subtracted from the average difference in volatilities within the event window.

Table 2

Average differences in implied standard deviations around quarterly expiration dates for call options on the Standard and Poor's 100 Index having different times to expiration.

Average differences are reported for each day in a nine-day window beginning four days before the quarterly expiration (−4) and ending four days after. The quarterly expiration is at τ_0. The dates τ_1, τ_2, and τ_3, respectively, indicate expirations one month, two months, and three months following the expiration date. The sample includes 15 quarterly expiration dates from June 1983 through December 1986. (t-statistics are in parentheses.)

Differences in implied standard deviations	Day in relation to quarterly expiration at day zero								
	−4	−3	−2	−1	0	1	2	3	4
$\sigma(\tau_0)-\sigma(\tau_1)$	1.5 (2.1)[a]	5.1 (7.3)[a]	7.2 (10.3)[a]	n.a.[b]	n.a.[b]	n.a.	n.a.	n.a.	n.a.
$\sigma(\tau_0)-\sigma(\tau_2)$	1.9 (2.7)[a]	5.9 (8.4)[a]	7.8 (11.1)[a]	n.a.[b]	n.a.[b]	n.a.	n.a.	n.a.	n.a.
$\sigma(\tau_0)-\sigma(\tau_3)$	2.4 (3.4)[a]	6.6 (9.4)[a]	9.0 (12.8)[a]	n.a.[b]	n.a.[b]	n.a.	n.a.	n.a.	n.a.
$\sigma(\tau_1)-\sigma(\tau_2)$	0.1 (0.1)	0.3 (0.4)	−0.1 (−0.1)	4.7 (6.7)[a]	3.0 (4.3)[a]	−1.0 (−1.4)	0.2 (0.3)	0.2 (0.3)	0.1 (0.1)
$\sigma(\tau_1)-\sigma(\tau_3)$	0.6 (0.9)	1.0 (1.4)	1.0 (1.4)	3.2 (4.6)[a]	4.9 (7.0)[a]	−1.2 (−1.7)	1.6 (2.3)[a]	0.7 (1.0)	−0.1 (−0.1)
$\sigma(\tau_2)-\sigma(\tau_3)$	0.5 (0.07)	0.7 (1.0)	0.9 (1.3)	−0.5 (−0.7)	1.4 (2.0)[a]	0.8 (1.1)	0.6 (0.9)	0.4 (0.6)	0.2 (0.3)

[a]Significant at the 5% level.
[b]Options due to expire in one day or less are excluded from the sample because the price of the option in relation to its exercisable value is low enough to make the bid–ask problem an issue.

volatility for the expiring option and the implied volatilities for options with longer times to expiration increases with the time to expiration of the longer option. These results are consistent with above-average market volatility at the quarterly expiration date, since the volatility at the expiration would have a much greater weight in determining the implied volatility for the expiring option than in determining the implied volatilities for longer options.

Above-average market volatility at the quarterly expiration date should have a greater impact on the implied volatility of an option with one month to expiration (at τ_1) than on the implied volatilities of the options with longer times to expiration (at τ_2 and τ_3). Averaging any temporary increase in volatility over a period as long as a month, however, might dilute the impact on implied volatility enough to cause the average differences in the implied volatilities of options spanning the quarterly expiration date to be very small. The last three rows of table 2 contain the average differences in implied volatilities for options whose expirations span the quarterly expiration date. The average differences in implied volatilities for any day in the event window except the day before expiration and the expiration day are (with one exception) not statistically significant. This result is consistent with either above-average volatility at the quarterly expiration or the null hypothesis of no increase in market volatility. For the day before expiration and the expiration date, however, the average differences between the implied volatility for the one-month option and the implied volatilities for the two longer options are statistically significant.

These results suggest that the increase in the volatility of the S&P 100 Index at the quarterly expiration date has both anticipated and unanticipated components. The anticipated increase seems to be reflected in the increase in the average difference between the implied volatilities for expiring options and options with longer times to expiration from day -4 through day -2. Examining the behavior of the implied volatilities for options which are not due to expire (at τ_0), we find small and statistically insignificant average differences in the implied volatilities of the one-month option and the longer options on these days. This is also consistent with an anticipated increase in volatility at the quarterly expiration because, for all practical purposes, such an increase is averaged out over the time to expiration for the one-month option. The dramatic increase in the average differences between the implied volatility of the one-month and those of the longer options on the day before the quarterly expiration date suggests an unexpected increase in market volatility.

The meaning of the significant difference in implied volatilities at the close of trading on the quarterly expiration date is less clear. One interpretation is that this difference reflects the anticipated volatility associated with the price reversals that tend to occur on the Monday following Friday's quarterly expiration (i.e., following the unwinding of futures-related arbitrage positions).

A second explanation is that the demand by options traders to close positions in expiring options and to open positions in the next expiration series on the quarterly expiration day (and perhaps on day −1 as well) creates a temporary upward bias in the prices of call options that is reflected in the estimates of implied volatilities.

The average differences between the implied volatilities of call options on the S&P 100 around nonquarterly expiration dates are reported in table 3. The sample includes 24 nonquarterly expirations from January 1984 through December 1986. The pattern of significance of the average differences in implied volatilities is similar to that around quarterly expirations. For example, the average differences between the implied volatility for the expiring option and the implied volatilities for the options spanning the expiration date are positive and statistically significant, although the magnitudes of the average differences are significantly smaller than their counterparts at quarterly expiration dates. The magnitude and statistical significance of the average differences in implied volatilities for options spanning the nonquarterly expiration dates are similar to those at the quarterly expiration for the day before and the day of the nonquarterly expiration.

The data presented in table 3 indicate that there is an increase in the volatility of the S&P 100 Index around the expiration of call options on the index that is independent of the unwinding of arbitrage positions in index futures contracts. As for quarterly expirations, the increase in the implied volatility of the expiring option series in relation to the implied volatilities for options with longer times to expiration is consistent with an anticipated increase in volatility at the nonquarterly expiration of index options. The evidence for an unexpected increase in volatility around the nonquarterly expirations is less convincing than that at the quarterly expiration date. In particular, the difference between the implied volatilities for the options expiring at τ_1 and τ_3 is statistically significant and increases monotonically from day −3 through the nonquarterly expiration date, which suggests that the increase in volatility through day −1 is anticipated.

Note the statistical significance of the average differences between the implied volatilities of the options spanning the expiration date at the close of trading on the nonquarterly expiration date. The magnitude of these differences is similar to that observed on quarterly expiration dates. Since there should be no futures-related price reversals on the Mondays following the nonquarterly expirations, the greater average implied volatility of the one-month option (with expiration at τ_1) at the close of trading on day zero is probably attributable to the demand by options traders to roll out of expiring contracts and into the next expiration series.

The results in this section indicate that the prices of call options on stock indexes anticipate increases in market volatility at the quarterly expiration of stock index futures contracts. Smaller increases in market volatility around the

Table 3

Average differences in implied standard deviations around nonquarterly expiration dates for call options on the Standard and Poor's 100 Index having different times to expiration.

Average differences are reported for each day in a nine-day window beginning four days before the nonquarterly expiration (−4) and ending four days after. The expiration date is τ_0. The dates τ_1, τ_2, and τ_3, respectively, indicate expirations one month, two months, and three months following the expiration date. The sample includes 24 nonquarterly expiration dates from January 1984 through November 1986. (*t*-statistics are in parentheses.)

Differences in implied standard deviations	Day in relation to nonquarterly expiration at day zero								
	−4	−3	−2	−1	0	1	2	3	4
$\sigma(\tau_0)-\sigma(\tau_1)$	0.8	2.2	2.7	n.a.[b]	n.a.[b]	n.a.	n.a.	n.a.	n.a.
	(1.4)	(4.0)[a]	(4.9)[a]						
$\sigma(\tau_0)-\sigma(\tau_2)$	1.5	2.9	4.0	n.a.[b]	n.a.[b]	n.a.	n.a.	n.a.	n.a.
	(2.7)[a]	(5.2)[a]	(7.2)[a]						
$\sigma(\tau_0)-\sigma(\tau_3)$	1.9	2.3	4.8	n.a.[b]	n.a.[b]	n.a.	n.a.	n.a.	n.a.
	(3.4)[a]	(4.2)[a]	(8.7)[a]						
$\sigma(\tau_1)-\sigma(\tau_2)$	0.6	0.6	0.6	1.8	3.4	−0.1	0.9	0.7	0.2
	(1.1)	(1.1)	(1.1)	(3.2)[a]	(6.1)[a]	(−0.2)	(1.6)	(1.3)	(0.4)
$\sigma(\tau_1)-\sigma(\tau_3)$	0.9	1.1	1.3	3.9	4.2	0.0	1.6	0.8	0.4
	(1.6)	(2.0)[a]	(2.3)[a]	(7.0)[a]	(7.6)[a]	(0.0)	(2.9)[a]	(1.4)	(0.7)
$\sigma(\tau_2)-\sigma(\tau_3)$	0.4	0.6	0.8	1.8	0.9	0.2	0.5	0.3	0.1
	(0.7)	(1.1)	(1.4)	(3.3)[a]	(1.6)	(0.4)	(0.9)	(0.5)	(0.2)

[a]Significant at the 5% level.
[b]Options due to expire in one day or less are excluded from the sample because the price of the option in relation to its exercisable value is low enough to make the bid–ask problem an issue.

nonquarterly expiration of options on individual stocks and stock indexes also appear to be anticipated. Significant differences between the implied volatilities of options spanning the immediate expiration date may be evidence that a portion of the increase in market volatility at both quarterly and nonquarterly expirations is unanticipated.[12] The extent to which unanticipated changes in market volatility occur before expiration dates is examined in the next section.

6. Tests of market efficiency

This section examines the time-series behavior of the returns on call options around both quarterly and nonquarterly expiration dates. Our purpose is to assess the significance of the unanticipated component of the *ex post* increase in market volatility documented by Stoll and Whaley (1987b). If call options fully anticipate the *ex post* increase in volatility, then returns should not reflect a systematic unanticipated volatility component. If unanticipated increases in market volatility occur before expiration dates, daily returns on option contracts before the expiration day will exceed their expected values. The behavior of the returns on stock index options is examined by performing an event study using the residual returns on options generated by the capital asset pricing model (CAPM).

The residual return for option i traded on day t in relation to expiration v is

$$\tilde{\varepsilon}_{it}^{v} = \tilde{r}_{it}^{v} - \left[r_{ft}^{v} + \beta_i \eta_{it}^{v} \left(\tilde{r}_{mt}^{v} - r_{ft}^{v} \right) \right], \tag{14}$$

where

$\tilde{r}_{mt}^{v}$ = realized daily return on the value-weighted NYSE and AMEX composite index from the Center for Research in Security Prices (CRSP) daily return file on day t in relation to expiration day v,

r_{ft}^{v} = daily yield on the U.S. Treasury bill with the expiration date closest to the expiration date of the option in relation to expiration day v,

$\tilde{r}_{it}^{v}$ = realized daily return on call option i at day t in relation to expiration day v,

η_{it}^{v} = option's price elasticity, $(\partial c/\partial I)(I/c)$, evaluated at the estimate of option i's implied volatility on day $t-1$,

β_i = market beta of the underlying index, which is assumed to be one for every index.

[12] Similar results are observed for call options on the Major Market Index. The results for the New York Stock Exchange Composite are less pronounced, but exhibit a similar pattern. This should be expected, since futures-related arbitrage is for the most part confined to a subset of the stocks included in the S&P 500. The results for the Major Market Index and the New York Stock Exchange Composite Index are available from the authors on request.

The residual returns are averaged across all expiration events using all options traded on each day t in relation to an expiration day. The average residual return across all options for day t, which we refer to as the average abnormal return (AAR) for day t, is defined by

$$AAR_t = \frac{1}{E} \sum_{v=1}^{E} \frac{1}{N_{vt}} \sum_{i=1}^{N_{vt}} \varepsilon_{it}^{v}, \tag{15}$$

where N_{vt} denotes the number of option contracts traded on day t in relation to the vth expiration. The test for the statistical significance is the ratio of the AAR to its estimated standard deviation,

$$TAAR_t = AAR_t/SE. \tag{16}$$

The standard deviation is computed using the residuals outside the event window in a manner analogous to the approach used for the standard deviation of volatility differences.

Table 4 shows the average abnormal returns for options on the S&P 100 Index, the Major Market Index and the NYSE Composite Index for a nine-day window surrounding quarterly and nonquarterly expiration days. For quarterly expiration days, the AARs for options on each of the three indexes are statistically insignificant from day -4 through day -2. On the day before the expiration date, there is a very large (38.7% to 85%) and statistically significant average abnormal return for options on both the S&P 100 and the Major Market Indexes. As might be expected, the AARs from the quarterly expiration date through the end of the expiration window are not statistically significant. The significant AAR at day -1 is consistent with an unexpected increase in market volatility. The AAR for options on the NYSE composite is large, but not statistically significant. This suggests that unanticipated market volatility at the quarterly expiration date primarily affects the stocks in the S&P 100 and Major Market Indexes, which are most closely linked to futures-related arbitrage activity.

The AARs around nonquarterly expirations are also consistent with an unanticipated increase in market volatility before the expiration date. Again, the AAR on the day before the expiration day is significantly positive for options on both the S&P 100 and Major Market Indexes. For nonquarterly expirations the expiration-day AAR for options on the Major Market Index is also statistically significant.

Our evidence is consistent with unanticipated increases in market volatility just prior to both quarterly and nonquarterly expirations. The evidence in section 5 suggests that the unwinding of futures-related arbitrage positions results in higher anticipated volatility before quarterly expirations than non-

Table 4

Average daily residual returns (in percent) on equally-weighted portfolios of call options on stock indexes for the nine-day windows beginning four days before and ending four days after quarterly and nonquarterly expirations. (t-statistics are in parentheses.)

Stock index	Day in relation to the expiration at day zero								
	−4	−3	−2	−1	0	1	2	3	4
(A) Quarterly expirations									
S&P 100[a]	0.1%	2.5%	0.7%	38.7%	5.0%	−0.2%	1.5%	−2.7%	−1.5%
	(0.03)	(0.77)	(0.22)	(11.95)[d]	(1.54)	(−0.06)	(0.46)	(−0.83)	(−0.46)
Major Market[b]	−0.8	8.2	0.4	85.0	10.2	3.8	−3.9	−3.7	−6.5
	(−0.09)	(0.97)	(0.05)	(10.00)[d]	(1.2)	(0.45)	(−0.46)	(−0.44)	(−0.77)
NYSE[c]	1.3	−1.7	5.2	31.6	2.6	0.6	19.2	0.4	−2.6
	(0.06)	(−0.07)	(0.22)	(1.34)	(0.11)	(0.03)	(0.81)	(0.02)	(−0.11)
(B) Nonquarterly expirations									
S&P 100[a]	0.7	0.9	7.7	46.7	2.5	4.5	6.3	−4.5	1.9
	(0.33)	(0.42)	(3.6)	(22.09)[d]	(1.18)	(2.13)[d]	(2.98)[d]	(−2.12)[d]	(0.89)
Major Market[b]	−1.3	7.9	−5.7	61.5	21.1	11.9	−5.8	−5.6	1.5
	(−0.18)	(1.11)	(−0.80)	(8.64)[d]	(2.96)[d]	(1.67)	(−0.81)	(−0.79)	(0.21)
NYSE[c]	−1.1	8.1	14.9	36.9	9.7	5.9	8.2	3.0	6.2
	(−0.03)	(0.21)	(0.38)	(0.95)	(0.25)	(0.15)	(0.21)	(0.08)	(0.16)

[a]The sample consists of 15 quarterly expirations and 24 nonquarterly expirations beginning with June 1983 and ending with December 1986.
[b]The sample consists of 12 quarterly expirations and 26 nonquarterly expirations beginning with July 1983 and ending with December 1986.
[c]The sample consists of 9 quarterly expirations and 22 nonquarterly expirations beginning with February 1985 and ending with December 1986.
[d]Significant at the 5% level.

quarterly expirations. The fact that increases in volatility also occur before nonquarterly expirations, however, suggests the existence of an independent source of market volatility, such as the covering of arbitrage positions in either index options or individual options and the underlying stocks.

7. Conclusion

We examine the volatility of the stock market around the quarterly expirations of stock index futures contracts and the nonquarterly expirations of stock index options, using estimates of the volatility implicit in the prices of stock index options. We find that option prices reflect increases in the volatility of the underlying stock indexes at both quarterly and nonquarterly expiration dates. The behavior of the implied volatilities for options spanning the expiration dates is consistent with an unexpected component to the increase in volatility. An analysis of the residual returns on index options in the nine-day window surrounding the quarterly and nonquarterly expiration dates shows significant and positive abnormal returns for call options on the S&P 100 and Major Market Indexes on the day before both quarterly and nonquarterly expirations. This evidence is consistent with an unexpected increase in market volatility around expiration dates.

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