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Reset Put Options: Valuation, Risk Characteristics, and an Application

by
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Abstract:

A reset put option is similar to a standard put option except that the exercise price is reset equal to the stock price on the pre-specified reset date if this stock price exceeds the original exercise price. In this paper we derive a valuation formula for a reset put option and present a range of comparative statics designed to highlight the differences between a reset put and a standard put. We also develop a numerical technique for valuing American-style reset puts. Finally, we apply our valuation results to assess the interest rate premiums embedded in the Geared Equity Investments offered by Macquarie Bank.

Keywords:

OPTION VALUATION, RESET OPTIONS, GEARED EQUITY INVESTMENTS

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Australian Journal of Management, Vol. 24, No. 1, June 1999, © The Australian Graduate School of Management

1. Introduction

Reset put options trade in a number of markets worldwide. These securities are like standard put options except that the exercise price is automatically reset at a higher level if the underlying stock price is above the original exercise price on the reset date. Sometimes reset put options are traded as separate securities. The International Finance Corporation's *Bear Market Reset Warrants* traded on the New York Stock Exchange and the Chicago Board Options Exchange are one such example.[1] At other times, reset puts are embedded in structured over-the-counter products. Macquarie Bank in Australia, for example, offers a *Geared Equity Investment* (GEI) that is, in essence, a collateralized loan plus a reset put. The purpose of this paper is to (a) develop valuation methods for reset put options, (b) use the valuation methods to examine the put's risk characteristics, and (c) apply the valuation methods to assess the interest rate premiums embedded in Macquarie Bank's GEI's.

2. Description and Valuation

Like a standard European-style put option, a European-style reset put option entitles its holder, upon exercise, to the difference between the exercise price and the underlying stock price. Unlike a standard put, however, a reset put has a stochastic exercise price. On the day the reset put is issued, the exercise price is set equal to the stock price. If, however, the stock price exceeds the original exercise price on a pre-specified future reset date, the exercise price is reset equal to the prevailing stock price.

To understand how to value a reset put with a single reset date specified at the time of issuance, first consider its terminal value at expiration, W_T:

$$W_T = \begin{cases} S_t - S_T, & \text{if } S_t > X, S_T \le S_t \\ X - S_T, & \text{if } S_t \le X, S_T \le X, \\ 0, \text{ if } (S_t > X \text{ and } S_T > S_t) \text{ or } (S_t \le X \text{ and } S_T > X) \end{cases} \tag{1}$$

where X is the original exercise price (which is the closing stock price on the date of issuance), S_t is the closing stock price on the reset date, and S_T is the closing stock price on the expiration date. The time subscripts t and T represent the reset date and the expiration date of the put, respectively. The first case in (1) is where the reset put is in-the-money at expiration ($S_T \le S_t$), after having its original exercise price reset at the stock price at time t because the stock price exceeded the original exercise price ($S_t > X$). The second case in (1) is where the reset put is in-the-money at expiration ($S_t \le X$), but where the exercise price was not reset because the stock price was below the original exercise price on the reset date ($S_T \le X$). The final case in (1) is where the put is out-of-the-money at expiration, independent of whether or not the put's exercise price was reset.

Assuming a riskless hedge may be formed between the reset put and the underlying stock, the reset put can be valued using risk-neutral valuation. Under risk-neutrality, the current value of the reset put equals the present value, at the

1. For a description of IFC's bear market reset warrants, see Gray and Whaley (1997).

risk-free rate, of its expected terminal value. The expected terminal value of the reset put, in turn, is the sum of the expected conditional terminal values, weighted by their probability of occurring, as defined in (1). Valuing the reset put, therefore, involves evaluating the terms of the right hand-side of (2) below:

$$\begin{aligned} W &= e^{-rT} E(S_t - S_T \mid S_t > X, S_T \leq S_t) \Pr(S_t > X, S_T \leq S_t) \\ &\quad + e^{-rT} E(X - S_T \mid S_t \leq X, S_T \leq X) \Pr(S_t \leq X, S_T \leq X), \\ &= W_1 + W_2. \end{aligned} \tag{2}$$

W_1 is the present value of the expected terminal value of the reset put conditional on the exercise price being reset and the reset put being in-the-money at expiration, weighted by the probability of this occurring. W_2 is the present value of the expected terminal value of the reset put conditional on the exercise price *not* being reset and the reset put being in-the-money at expiration, weighted by the probability of this occurring. In all cases, expectations and probabilities are under the risk-neutral measure.

Under the Black-Scholes (1973) framework, the price of the stock underlying the option is assumed to follow geometric Brownian motion,

$$\frac{dS}{S} = \mu dt + \sigma dz,$$

where μ is the expected return on the stock, σ is the standard deviation of the stock return, and z is a Wiener process. Among other things, this implies that stock returns are serially independent and that the stock price is lognormally distributed at any future time τ. We now apply this assumption to evaluate W_1 and W_2 in (2).

Focusing first on W_1, the conditional expectation can be simplified by separating the terms in the difference:

$$\begin{aligned} W_1 &= E[S_t - S_T \mid S_t > X, S_T \leq S_t] \Pr[S_t > X, S_T \leq S_t] e^{-rT} \\ &= E[S_t \mid S_t > X, S_T \leq S_t] \Pr[S_t > X, S_T \leq S_t] e^{-rT} \\ &\quad - E[S_T \mid S_t > X, S_T \leq S_t] \Pr[S_t > X, S_T \leq S_t] e^{-rT} \end{aligned} \tag{3}$$

By the Markov property of stock prices,

$$E[S_t \mid S_t > X, S_T \leq S_t] = E[S_t \mid S_t > X].$$

Since stock returns are independent, knowing that the stock price fell between time t and time T tells us nothing about the level of the stock price at time t. That is, the only useful piece of information in the conditioning set is that $S_t > X$.

The independence of returns also implies that the probability expression in W_1 can be written as

$$\Pr(S_t > X, S_T \le S_t) = \Pr(S_t > X)\Pr(S_T \le S_t).$$

This equation describes the probability that the stock price will be above X at time t **and** will subsequently fall between time t and time T. Since these two events are independent (by the Markov property of stock prices), the probability of the joint event is simply the product of the two individual probabilities. Hence, (3) may be written as:

$$\begin{aligned} W_1 &= E[S_t - S_T \mid S_t > X, S_T \le S_t]\Pr[S_t > X, S_T \le S_t]e^{-rT} \\ &= E[S_t \mid S_t > X]\Pr[S_t > X]\Pr[S_T \le S_t]e^{-rT} \\ &-E[S_T \mid S_t > X, S_T \le S_t]\Pr[S_t > X]\Pr[S_T \le S_t]e^{-rT} \end{aligned} \tag{4}$$

From the properties of a lognormal distribution, the two terms on the right-hand side of (4) can be expressed as

$$E[S_t \mid S_t > X]\Pr[S_t > X]\Pr[S_T \le S_t]e^{-rT} = Se^{-dt}N(a_1)N(-c_2)e^{-r(T-t)}$$

and

$$E[S_T \mid S_t > X, S_T \le S_t]\Pr[S_t > X]\Pr[S_T \le S_t]e^{-rT} = Se^{-dT}N(a_1)N(--c_1)$$

where

$$a_1 = \frac{\ln(S/X) + (r - d + 0.5\sigma^2)t}{\sigma\sqrt{t}}, a_2 = a_1 - \sigma\sqrt{t},$$
$$c_1 = \frac{(r - d + 0.5\sigma^2)(T-t)}{\sigma\sqrt{T-t}}, c_2 = c_1 - \sigma\sqrt{T-t},$$

r is the riskless rate of interest, d is the stock's dividend yield rate, $N(a)$ and is a cumulative univariate normal distribution function with upper integral limit a. Therefore, W_1 can be written as risk-free rate, of its expected terminal value. The expected terminal value of the reset put, in turn, is the sum of the expected conditional terminal values, weighted by their probability of occurring, as defined in (1). Valuing the reset put, therefore, involves evaluating the terms of the right hand-side of (2) below:

$$\begin{aligned} W &= e^{-rT}E(S_t - S_T \mid S_t > X, S_T \le S_t)\Pr(S_t > X, S_T \le S_t) \\ &\quad + e^{-rT}E(X - S_T \mid S_t \le X, S_T \le X)\Pr(S_t \le X, S_T \le X), \\ &= W_1 + W_2. \end{aligned} \tag{2}$$

W_1 is the present value of the expected terminal value of the reset put conditional on the exercise price being reset and the reset put being in-the-money at expiration, weighted by the probability of this occurring. W_2 is the present value of the expected terminal value of the reset put conditional on the exercise price *not* being reset and the reset put being in-the-money at expiration, weighted by the probability of this occurring. In all cases, expectations and probabilities are under the risk-neutral measure.

Under the Black-Scholes (1973) framework, the price of the stock underlying the option is assumed to follow geometric Brownian motion,

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where μ is the expected return on the stock, σ is the standard deviation of the stock return, and z is a Wiener process. Among other things, this implies that stock returns are serially independent and that the stock price is lognormally distributed at any future time τ. We now apply this assumption to evaluate W_1 and W_2 in (2).

Focusing first on W_1, the conditional expectation can be simplified by separating the terms in the difference:

$$\begin{aligned} W_1 &= E[S_t - S_T \mid S_t > X, S_T \le S_t]\Pr[S_t > X, S_T \le S_t]e^{-rT} \\ &= E[S_t \mid S_t > X, S_T \le S_t]\Pr[S_t > X, S_T \le S_t]e^{-rT} \\ &\quad - E[S_T \mid S_t > X, S_T \le S_t]\Pr[S_t > X, S_T \le S_t]e^{-rT} \end{aligned} \tag{3}$$

By the Markov property of stock prices,

$$E[S_t \mid S_t > X, S_T \le S_t] = E[S_t \mid S_t > X].$$

$$W_1 = Se^{-dt}N(a_1)N(-c_2)e^{-r(T-t)} - Se^{-dT}N(a_1)N(-c_1). \tag{5}$$

Next, break up the second piece of the original formula (2):

$$\begin{aligned} W_2 &= E[X - S_T \mid S_t < X, S_T \le X]\Pr[S_t < X, S_T \le X]e^{-rT} \\ &= X\Pr[S_T < X, S_T \le X]e^{-rt} \\ &\quad - E[S_T \mid S_t < X, S_T \le X]\Pr[S_t < X, S_T \le X]e^{-rT}. \end{aligned} \tag{6}$$

Now

$$\Pr[S_t < X, S_T \le X] = N_2\left(-a_2, -b_2, \sqrt{t/T}\right)$$

where

$$b_1 = \frac{\ln(S/X) + \left(r - d + 0.5\sigma^2\right)T}{\sigma\sqrt{T}}, b_2 = b_1 - \sigma\sqrt{T},$$

where N_2(a, b; ρ) is a cumulative bivariate normal distribution function with upper integral limits a and b and correlation coefficient ρ. Therefore (6) may be written as

$$\begin{aligned} W_2 &= E[X - S_T \mid S_t < X, S_T \le X]\Pr[S_t < X, S_T \le X]e^{-rT} \\ &= Xe^{-rT}N_2\left(-a_2, -b_2, \sqrt{t/T}\right) - Se^{-dT}N_2\left(-a_1, -b_1, \sqrt{t/T}\right). \end{aligned}$$

Hence, the European-style reset put option formula is

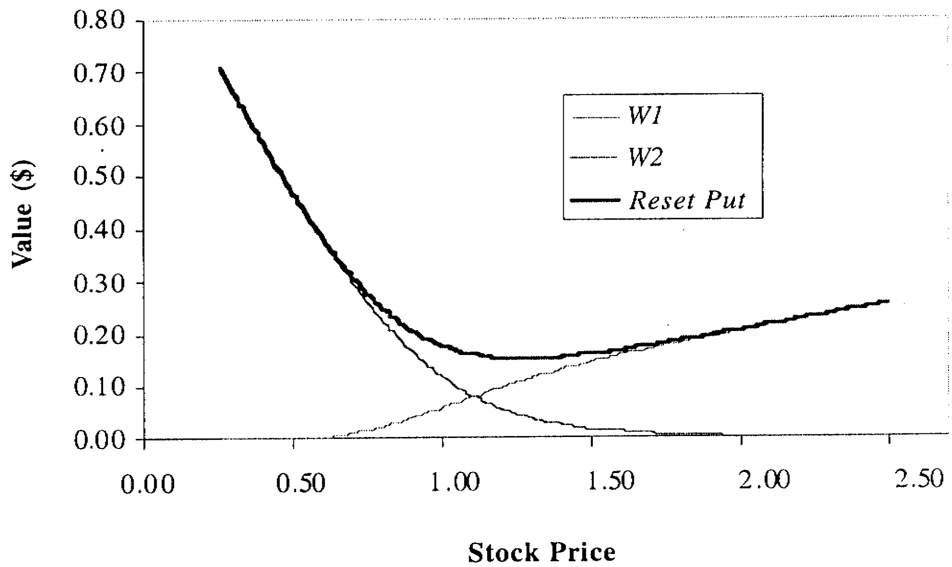
$$\begin{aligned} W &= W_1 + W_2 \\ &= Se^{-dt}N(a_1)N(-c_2)e^{-r(T-t)} - Se^{-dT}N(a_1)N(-c_1) \\ &= Xe^{-rT}N_2\left(-a_2, -b_2, \sqrt{t/T}\right) - Se^{-dT}N_2\left(-a_1, -b_1, \sqrt{t/T}\right). \end{aligned} \tag{7}$$

Figure 1 illustrates the values of the two components of (7). The figure was prepared assuming the reset put has an exercise price of 1 and a time to expiration of 365 days. The reset put's exercise price may be reset in 182 days should the stock price exceed 1. The interest rate is assumed to be 5% p.a., the dividend yield on the stock is 2% p.a., and the volatility rate of the stock is 40% p.a. As the stock price rises, the value of the put with the original exercise price, W_2, falls. At the same time, as the stock price rises, both the likelihood that the exercise price will be reset and the level of the expected exercise price rise. Hence, the value of W_1

rises. As the figure shows, the two components of the reset put premium have offsetting effects. This U-shaped behavior is due to a tradeoff between the relative likelihood of the two events that would result in the reset put yielding a positive payoff at maturity. Recall that a reset put will yield a positive payoff if (a) the stock price falls and finishes below the original exercise price (the "no-reset payoff" W_2), or (b) the stock price rises, the exercise price is reset, then the stock price falls and finishes below the reset exercise price (the "reset payoff" W_1). As the stock price begins to rise, the likelihood of receiving a no-reset payoff is diminished and this effect dominates the marginal increase in the expected reset payoff. As the stock price rises further, the expected reset payoff continues to increase and becomes the dominant effect.

Figure 1

Components of Reset Put Value



Note: This figure shows the reset and standard put values for a range of current stock prices. For both options the original exercise price is $1, the time to maturity is one year (365 days), the volatility of the stock is 40% p.a., the riskless interest rate is 5% p.a., the dividend yield on the stock is 2% p.a. There is a single reset date after six months (182 days).

3. Risk Characteristics

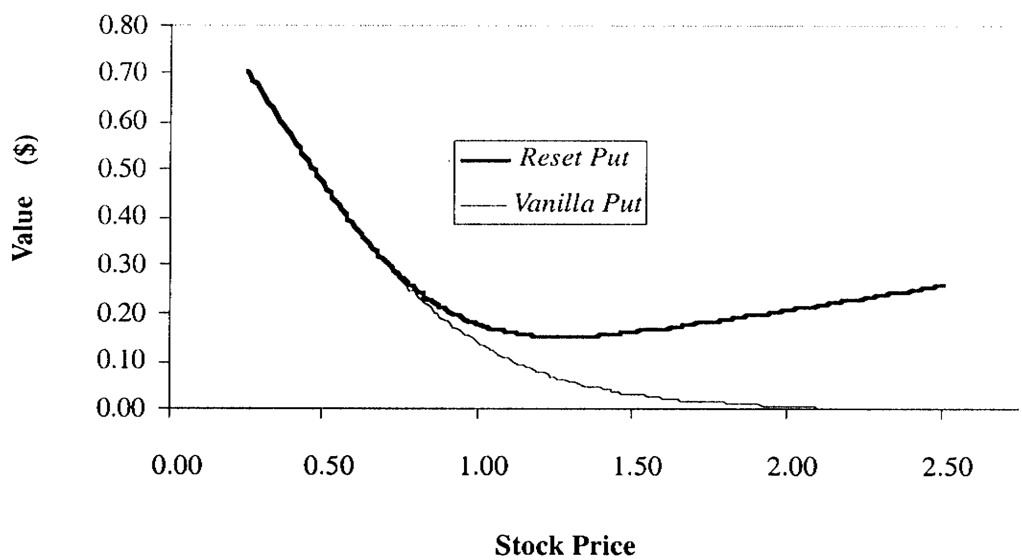
To illustrate the risk characteristics of the reset put and compare the reset put with a standard put, we examine the change in option values as the valuation parameters change. We continue to assume that the reset put has an exercise price of 1 and a time to expiration of 365 days. The reset put's exercise price may be reset in 182 days should the stock price exceed 1. The interest rate is assumed to be 5% p.a., the dividend yield on the stock is 2% p.a., and the volatility rate of the stock is 40% p.a.

Figure 2 illustrates the difference between the value of the reset put and the value of a standard put. The standard put value is computed using the Black-Scholes (1973) put option valuation formula,

$$p = Xe^{-rT}N_2(-b_2) - Se^{-dT}N_2(-b_1), \tag{8}$$

where b_1 and b_2 are as previously defined. As the stock price falls, the value of the reset put approaches the value of the standard put. This should not be surprising in the sense that, with a reduction in share price, the chances of the reset put's exercise price being reset are diminished. On the other hand, as the stock price increases, the value of the reset put initially falls but then later rises whereas the standard put value falls to zero.

Figure 2
Reset and Vanilla Put Values



Note: This figure shows the reset and standard put values for a range of current stock prices. For both options the original exercise price is $1, the time to maturity is one year (365 days), the volatility of the stock is 40% p.a., the riskless interest rate is 5% p.a., the dividend yield on the stock is 2% p.a. For the reset put, there is a single reset date after six months (182 days).

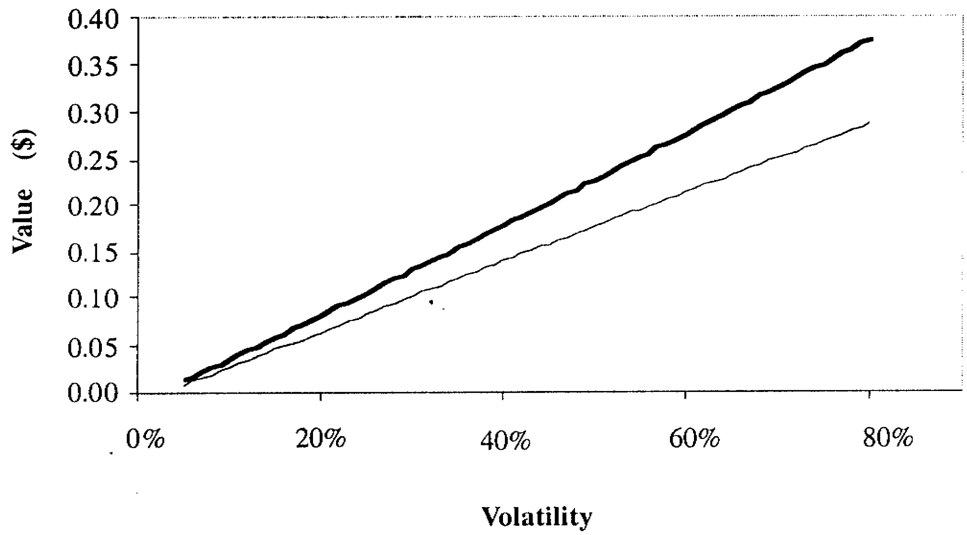
Figure 3 illustrates the effect that changes in volatility have on a reset put and a standard put.[2] Except for volatility, the valuation parameters are the same as in figures 1 and 2. The figure clearly demonstrates that the value of the reset put is more sensitive to changes in volatility. As volatility increases, both the reset put and the standard put become more valuable since there is more chance of the

2. Due to the complexity of the reset put valuation formula, we compute all partial derivatives numerically rather than analytically.

option finishing well in-the-money. In addition, the reset put becomes even more valuable, since higher volatility also means higher probability of the exercise price being reset, in which case the payoff at maturity will be greater than under a standard put.

Figure 3

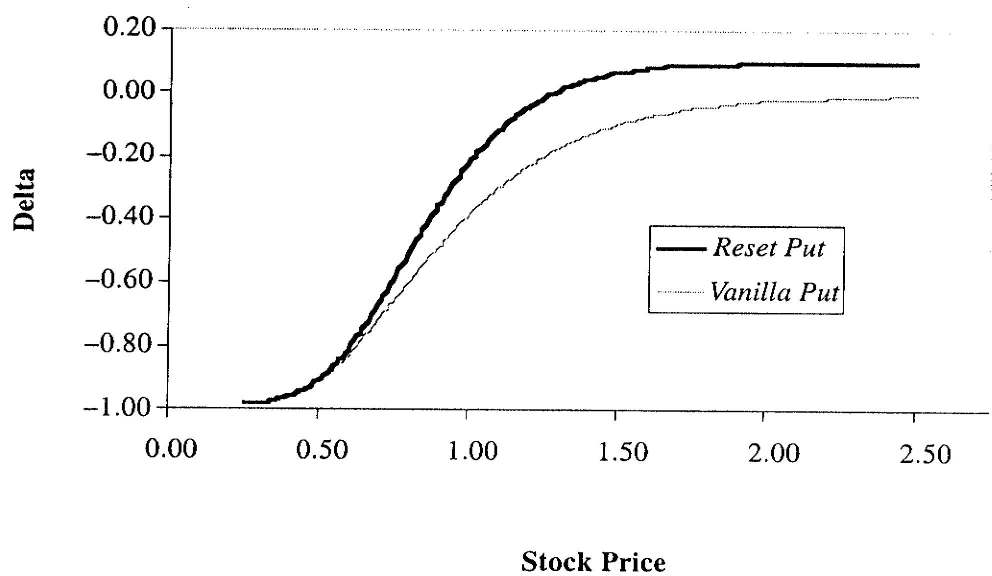
Reset and Vanilla Put Values and Volatility



Note: This figure shows the reset and standard put values for a range of volatilities. For both options the original exercise price is $1, the time to maturity is one year (365 days), the current stock price is $1, the riskless interest rate is 5% p.a., the dividend yield on the stock is 2% p.a. For the reset put, there is a single reset date after six months (182 days).

Figure 4 shows the relation between the reset and standard put deltas and the underlying stock price. Put option deltas indicate the number of shares that must be purchased in order to hedge an option position. Note that the standard put delta is zero for any stock price beyond about $2. In this range, it is almost inevitable that the put will finish out of the money so the put is essentially worthless. Moreover, small changes in the stock price have little effect on the value of this put. Conversely, the reset put delta can become positive. As the stock price continues to rise, reset is inevitable in which case the reset put is really just a claim to receive an at-the-money standard put on the reset date. Since this standard put has an exercise price that is equal to the stock price on the reset date, its value increases as the stock price increases. That is, an at-the-money standard put struck at $3 is more valuable than an at-the-money put struck at $2. Hence, as the stock price increases the reset put becomes the right to receive an ever more valuable at-the-money standard put with time to maturity equal to the time between the reset date and the original maturity.

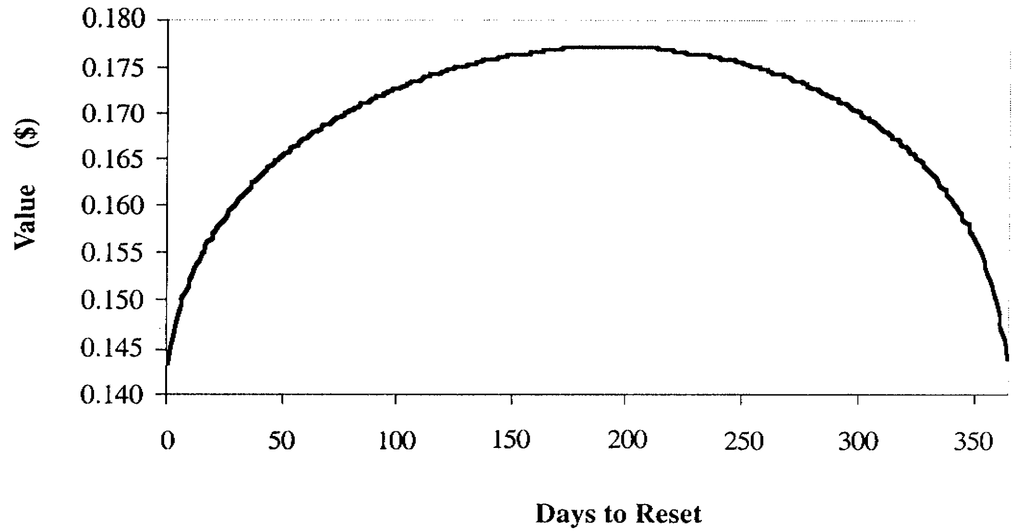
Figure 4
Reset and Vanilla Put Deltas



Note: This figure shows the reset and standard put deltas for a range of current stock prices. For both options the original exercise price is \$1, the time to maturity is one year (365 days), the volatility of the stock is 40% p.a., the riskless interest rate is 5% p.a., the dividend yield on the stock is 2% p.a. For the reset put, there is a single reset date after six months (182 days).

Figure 5 demonstrates the value of a reset put as a function of the timing of the reset date. The reset feature has little value when the reset date is close to the origination date since the stock has limited opportunity to appreciate in value before the exercise price is reset. Similarly, placing the reset date close to the maturity date limits its value since the stock has limited opportunity to fall in value below the reset exercise price. Hence, there is an internal optimum reset date. In Figure 5, the optimal reset date is after 192 days, where the value of the reset put is \$0.1772.

Figure 5
Reset Put Value and Reset Date



Note: This figure shows the reset put values for a range of reset dates. The original exercise price is \$1, the time to maturity is one year (365 days), the volatility of the stock is 40% p.a., the riskless interest rate is 5% p.a., the dividend yield on the stock is 2% p.a.

For a range of volatilities and stock prices, Table 1 quantifies the "reset premium", which we define as the premium that must be paid for a reset put (over and above the price of an otherwise identical standard put). Two features are apparent from the table.

First, as the stock price increases above the original exercise price, the reset put becomes much more valuable than a standard put. This is because it becomes more likely that the reset feature will be activated so that the reset put will become an at-the-money instrument on the reset date. Conversely, the standard put becomes likely to expire out-of-the-money and consequently has a very low value. Second, this effect is exacerbated when volatility is high. Higher volatility means a greater range of possible movements in the price of the underlying stock. Hence, when volatility is high, the reset put has a greater chance of being reset with a high exercise price. That is, when volatility is high, the reset put has an increased chance of becoming an at-the-money standard put with high exercise price on the reset date. Since the value of an at-the-money standard put is increasing in the exercise price, high volatility increases the value of the reset feature.

Table 1

Reset Premiums

	Volatility						
Stock price	10%	20%	30%	40%	50%	60%	70%
0.50	0.0000	0.0000	0.0000	0.0002	0.0011	0.0029	0.0057
0.75	0.0000	0.0004	0.0030	0.0078	0.0142	0.0219	0.0304
1.00	0.0081	0.0174	0.0273	0.0381	0.0496	0.0619	0.0750
1.25	0.0258	0.0511	0.0681	0.0833	0.0985	0.1144	0.1311
1.50	0.0312	0.0707	0.1027	0.1275	0.1494	0.1703	0.1914
1.75	0.0364	0.0837	0.1282	0.1654	0.1967	0.2249	0.2519
2.00	0.0416	0.0958	0.1493	0.1978	0.2396	0.2766	0.3108

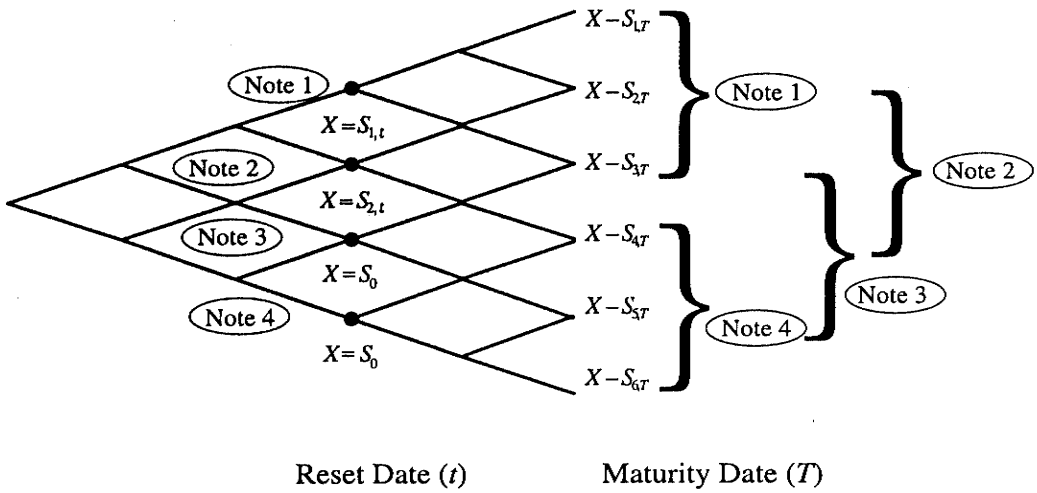
Note: This table shows the reset premium for a range of volatilities and stock prices. The reset premium is the premium that must be paid for a reset put over and above the price of an otherwise identical standard put. For both options the original exercise price is \$1, the time to maturity is one year (365 days), the riskless interest rate is 5% p.a., the dividend yield on the stock is 2% p.a. For the reset put, there is a single reset after six months (182 days). For example, for stock volatility of 40% an at-the-money \$1 reset put will cost 3.81 cents more than an otherwise identical standard put.

4. American-style Reset Puts

The valuation equation and the discussion above presumes that the reset put is a European-style instrument. The European-style feature permits an analytical valuation formula to be derived. With American-style reset puts such as Macquarie Bank's Geared Equity Investment, analytical solutions are generally not possible and numerical methods must be used. The most popular form of American-style option valuation approximation is the binomial method introduced by Cox, Ross, and Rubinstein (1979) and Rendleman and Bartter (1979). This procedure can be adapted to value American-style reset puts.

Applying the binomial lattice procedure to value American-style options is described in most standard options textbooks (e.g. Stoll & Whaley 1993, pp. 202–6; Hull 1996, pp. 201–4). The modifications necessary to value reset puts are as follows. First, the binomial lattice is set up such that the reset put's life is assumed to end at the reset date. Second, in place of using the option's terminal boundary condition to value the option at each asset price node, use another binomial lattice. For each of the asset price nodes in excess of the original exercise price, the binomial lattice procedure is used to value an at-the-money American-style standard put. The standard put is at-the-money since the reset put's exercise price is reset equal to the prevailing asset price. Standard put option valuation is appropriate at this point in time since after the reset date the reset put is identical to a standard put. For each of the asset price nodes below the original exercise price, the binomial lattice is used to value an American-style standard put with an exercise price of X (the original exercise price). Finally, with the reset put values at all of the nodes on the reset date identified, the valuation procedure proceeds backward through time, one step at a time, with new nodes being computed as the present value of the expected value of the two nodes directly in front. Each of these newly computed values are checked for the prospect of early exercise, and, if early exercise is optimal, the exercise proceeds replace the computed value at that node. The iterative procedure proceeds backwards through time until the current value of the reset put is identified. This procedure is illustrated in Figure 6.

Figure 6
Binomial Valuation of American-style Reset Option



Note 1: On the reset date, the stock price has risen, so the exercise price is reset to the current stock price $S_{1,t}$ where the notation signifies that this is the stock price at the top node on date t. The value of the option at this point proceeds in the standard way. From the top node at date t, three nodes can be reached at date T (nodes 1, 2, and 3). The payoff at each node will be the stock price at that node less the appropriate exercise price.

Note 2: On the reset date, the stock price has risen, so the exercise price is reset to the current stock price $S_{2,t}$ where the notation signifies that this is the stock price at the second top node on date t. The value of the option at this point proceeds in the standard way. From the second top node at date t, three nodes can be reached at date T (nodes 2, 3, and 4). The payoff at each node will be the stock price at that node less the appropriate exercise price. Note that the exercise price used for nodes 2 and 3 will be different from that employed under Note 1 above.

Note 3: On the reset date, the stock price has fallen, so the exercise price is not reset and remains equal to the stock price at date *0*, when the contract was entered into. The value of the option at this point proceeds in the standard way. From the second bottom node at date t, three nodes can be reached at date T (nodes 3, 4, and 5). The payoff at each node will be the stock price at that node less the appropriate exercise price. Note that the exercise price used for nodes 3 and 4 will be different from that employed under Note 2 above.

Note 4: On the reset date, the stock price has fallen, so the exercise price is not reset and remains equal to the stock price at date *0*, when the contract was entered into, as in Note 3 above.

5. Applying the Model to Macquarie Bank's Geared Equity Investment Product

Macquarie Bank is currently marketing two types of geared investment. With their *Margin Lending* facility, Macquarie provides an investor with a loan, the proceeds of which are used to purchase the shares of one of a number of Australian companies. The investor owns the shares and receives dividends paid by the company, and Macquarie holds the shares as collateral for the loan. Investors have the option of locking in a fixed rate of interest for the period of the loan and paying interest one year in advance to accelerate tax benefits. The current one-year fixed rate (as at 3 June 1998) for interest payable in advance is 7.85% p.a.

In addition to this standard form of gearing, Macquarie now markets a product called the *Geared Equity Investment* (GEI). The terms of this product are like those of the *Margin Lending* agreement, except that, in addition to lending the funds, Macquarie insures the investor against any share price decline. Investors choose between one, three, and five year maturity periods. For the three and five year maturities, the put option has an optional reset feature that automatically resets the exercise price to the closing stock price on a specified future date should the stock price exceed the original exercise price. The investor must choose the reset date at origination. Available reset dates are 31 January and 31 July each year between origination and maturity. There is no secondary market for the GEI's, hence prices can only be observed at the time of origination. Macquarie quotes the prices in terms of an interest rate that is applied to the loan that is used to purchase the shares.

In effect, a GEI consists of a standard *Margin Lending* loan plus an at-the-money put option (possibly with a reset feature). The premium being charged for the put can be computed by decomposing the interest rate charged for the GEI into its component pieces. If an investor were to borrow money for one year to purchase BHP shares using the standard *Margin Lending* facility, for example, interest of 7.85% would be payable at origination. Alternatively, if the investor were to use the GEI, a rate of 19.98% would apply. The GEI is a package of a loan and a put option. Since the loan rate is 7.85%, the cost of the put option is 12.13%. That is, the investor is being charged 12.13% of the price of the underlying stock for a one-year at-the-money put option.

Table 2 documents the premium for the put feature that is implicit in the interest rates quoted by Macquarie Bank for its one-year GEI product. Note that for the one-year GEI product the embedded put option has no reset feature. Therefore, the product essentially amounts to a loan plus a one-year standard put option struck at-the-money.

The first column in Table 2 contains the standard European-style put premium as a percentage of the underlying stock price for an at-the-money put with one year to maturity and no reset feature. Although early exercise of the put component of the GEI is allowed, fees and penalties are substantial making rational early exercise of the put feature unlikely.[3] The interest rate used in

3. In the event of early exercise, the investor is required to pay "all costs incurred by the bank in obtaining an appropriate form of risk management" which can "represent about 5% of the loan principal for each remaining year of the loan" (GEI Information Booklet, p. 20). In addition, the investor must pay a fee equal to one month's interest on the amount to be repaid, plus any costs involved in unwinding any fixed interest rate arrangement, government charges, stamp duty, and brokerage.

computing option values is the implied yield (continuously compounded) on a one-year government bond and is obtained from *The Australian Financial Review* web site, www.tradingroom.com.au. The dividend yield rate over the ensuing year is assumed to be the same as the previous year. In all cases we obtain the implied volatility from at-the-money standard puts trading on the Australian Stock Exchange (ASX) from the *Australian Market Quote* system.

Table 2

One-Year Geared Equity Investments with No Reset on 3 June 1998

	(1)	(2)	(3)	(4)
	Standard Put Premium	Actual Put Premium	Maximum Tax Benefit	Actual Put Premium After Tax
Amcor Limited	9.03%	12.44%	0.46%	11.98%
ANZ Banking Group	11.33%	13.44%	0.49%	12.95%
Broken Hill Proprietary Co Ltd	9.48%	12.13%	0.44%	11.69%
Brambles Industries	8.06%	11.66%	0.43%	11.23%
Boral Limited	11.27%	11.63%	0.43%	11.20%
Commonwealth Bank of Australia	8.34%	12.62%	0.46%	12.16%
Coca-Cola Amatil	13.61%	18.47%	0.68%	17.79%
Coles Myer Limited	8.83%	12.12%	0.44%	11.68%
CSR Limited	11.66%	12.81%	0.47%	12.34%
Foster's Brewing Group Ltd	8.60%	12.20%	0.45%	11.75%
Lend Lease Corporation Ltd	10.11%	15.21%	0.56%	14.65%
MIM Holdings Ltd	15.74%	14.58%	0.53%	14.05%
National Australia Bank	8.35%	13.08%	0.48%	12.60%
The News Corporation Ltd	8.23%	10.44%	0.38%	10.06%
Oil Search Limited	14.90%	19.21%	0.70%	18.51%
Qantas Airways	12.09%	13.90%	0.51%	13.39%
Westpac Banking Corporation	8.47%	11.56%	0.42%	11.14%
Average	10.48%	13.38%	0.49%	12.89%

Note: Column (1) shows the standard put premium as a percentage of the underlying stock price. Column (2) shows the put premium implied from quoted GEI interest rates. Column (3) provides the maximum tax benefit as a result of interest prepayment (assuming a 48.5% marginal tax rate), and column (4) shows the put premium of the GEI after tax relative to the value of a standard ASX put option. All calculations are based on market interest rates, implied volatilities from at-the-money options trading on the ASX, and historical dividend yields.

The second column contains the implied premium for the one-year, at-the-money put with no reset feature embedded in the GEI "indicative" rates quoted by Macquarie on 3 June 1998 (see the Macquarie web site www.macquarie.com.au). This value is obtained by subtracting the standard *Margin Lending* loan rate of 7.85% p.a. from the quoted GEI rate. Table 2 shows that the put premium implicit in the GEI is higher than the premium in the corresponding ASX puts for all companies examined except MIM Holdings. Indeed, the average premium for the put options embedded in the GEI (13.38%) is greater than the corresponding average ASX put premium (10.48%) by nearly 3% of the underlying stock price.

One possible reason for the GEI's higher premium relative to the ASX put options is differential tax treatment. Although both products require the payment of the put premium in advance, under the terms of the GEI the put premium is considered to be part of the interest payment and is immediately deductible for tax purposes. With a standard put on the ASX, the put premium is considered to be a capital payment and generates a tax benefit only at maturity. If the put option generates a maturity payoff less than its initial cost, this is considered to be a capital loss for tax purposes and can only be used to offset current or future capital gains. More generally, for the GEI, the put premium is deductible at origination, and, for a standard ASX put, the premium can be used to reduce capital gains at maturity. If we assume that the investor has capital gains against which any future capital losses can be offset, the tax advantage (*TA*) of immediate deductibility versus a future reduction in capital gains tax is

$$TA = P\tau\left(1 - e^{-rT}\right)$$

where: P is the put premium;
τ = the investor's marginal tax rate (e.g. in Australia, the highest personal tax rate is 48.5% of ordinary income and capital gains); and
r = the opportunity cost of funds for the investor.

These assumptions seem reasonable given that the minimum GEI loan amount is \$100,000. Column (3) of Table 2 contains the maximum tax benefit accorded each GEI, and column (4) contains the put premium after the tax advantage of the GEI has been taken into account. Even after allowing for the GEI's tax advantage, the GEI put premium is consistently greater than the corresponding premium on ASX put options. The average GEI put premium after tax is 12.89%, still nearly 240 basis points higher than comparable ASX puts.

Table 3 documents the premium for the put feature that is implicit in the interest rates quoted by Macquarie Bank for its three-year GEI product with no reset feature. This product essentially amounts to a loan plus a three-year standard put option struck at-the-money. Rather than being paid in a lump sum at origination, GEI investors effectively pay the put premium in three equal installments. One installment is paid at origination, one after one year, and one after two years. Each installment is equal to the total rate quoted for the GEI minus the corresponding *Margin Lending* rate. To compute the total present value of the put premium, we discount the two future installments using the yield on government securities with maturities of one and two years, respectively. In this way, all figures in Table 3 are expressed as a percentage of the underlying stock price.

The results of Table 3 are consistent with Table 2. Even after allowing for the tax advantages, the GEI put premiums are consistently greater than the corresponding premium on ASX put options. The average difference is now 8.30% (i.e. the average after tax put premium is 23.75% for the GEI's and 15.45% for the ASX options), reflecting the fact that these are longer term GEI's. On an annualized basis, the excess interest rate premium of the three-year GEI with no reset is about 2.9%,[4] slightly higher than the 2.4% reported for the one-year GEI.

Table 3
Three-Year Geared Equity Investment with No Reset on 3 June 1998

	(1)	(2)	(3)	(4)
	Standard Put Premium	Actual Put Premium	Maximum Tax Benefit	Actual Put Premium After Tax
Amcor Limited	13.88%	24.88%	2.53%	22.34%
ANZ Banking Group	17.10%	27.85%	2.83%	25.02%
Broken Hill Proprietary Company Ltd	13.95%	24.99%	2.54%	22.45%
Brambles Industries	11.22%	22.96%	2.34%	20.62%
Boral Limited	17.09%	24.02%	2.44%	21.57%
Commonwealth Bank of Australia	12.95%	25.79%	2.62%	23.17%
Coca-Cola Amatil	19.70%	31.42%	3.20%	28.23%
Coles Myer Limited	12.76%	22.05%	2.24%	19.80%
CSR Limited	17.68%	25.53%	2.60%	22.94%
Foster's Brewing Group Ltd	12.41%	23.96%	2.44%	21.52%
Lend Lease Corporation Ltd	14.69%	30.19%	3.07%	27.12%
MIM Holdings Ltd	23.40%	27.11%	2.76%	24.35%
National Australia Bank	12.50%	25.79%	2.62%	23.17%
The News Corporation Ltd	10.91%	21.07%	2.14%	18.93%
Oil Search Limited	21.29%	37.91%	3.86%	34.06%
Qantas Airways	18.57%	30.34%	3.09%	27.25%
Westpac Banking Corporation	12.53%	23.59%	2.40%	21.19%
Average	15.45%	26.44%	2.69%	23.75%

Note: Column (1) shows the standard put premium as a percentage of the underlying stock price. Column (2) shows the put premium implied from quoted GEI interest rates. Column (3) provides the maximum tax benefit as a result of interest prepayment (assuming a 48.5% marginal tax rate), and column (4) shows the put premium of the GEI after tax relative to a standard ASX put option. All calculations are based on market interest rates, implied volatilities from at-the-money options trading on the ASX, and historical dividend yields.

4. The present value of three instalments of 8.30%, so each payment is about 2.88%.

Table 4 documents the premium for the put feature that is implicit in the interest rates quoted by Macquarie Bank for its three-year GEI product with a single reset. Model prices are computed using data for 3 June 1998. Although the GEI allows the investor to choose among reset dates, we use 31 January 2000 because that reset date maximizes the value of the GEI. (Recall the earlier discussion showing the effect that reset date has on the value of the GEI.)

Table 4

Three-Year Geared Equity Investment with One Reset on 3 June 1998

	(1)	(2)	(3)	(4)	(5)	(6)
	Standard Put Premium	Reset Put Premium	Actual Put Premium	Max. Tax Benefit	Actual Put Premium After Tax	Reset Premium
Amcor Limited	13.88%	14.64%	25.59%	2.60%	22.99%	0.76%
ANZ Banking Group	17.10%	18.68%	28.56%	2.91%	25.66%	1.58%
Broken Hill Proprietary Company Ltd	13.95%	15.05%	25.71%	2.62%	23.09%	1.10%
Brambles Industries	11.22%	12.11%	23.67%	2.41%	21.27%	0.89%
Boral Limited	17.09%	18.62%	24.73%	2.52%	22.22%	1.53%
Commonwealth Bank of Australia	12.95%	13.48%	26.51%	2.70%	23.81%	0.53%
Coca-Cola Amatil	19.70%	22.69%	32.14%	3.27%	28.87%	2.99%
Coles Myer Limited	12.76%	13.74%	22.76%	2.32%	20.44%	0.98%
CSR Limited	17.68%	19.34%	26.25%	2.67%	23.58%	1.66%
Foster's Brewing Group Ltd	12.41%	13.32%	24.68%	2.51%	22.16%	0.91%
Lend Lease Corporation Ltd	14.69%	16.10%	30.91%	3.15%	27.76%	1.41%
MIM Holdings Ltd	23.40%	27.02%	27.82%	2.83%	24.99%	3.62%
National Australia Bank	12.50%	13.18%	26.51%	2.70%	23.81%	0.68%
The News Corporation Ltd	10.91%	12.08%	21.79%	2.22%	19.57%	1.17%
Oil Search Limited	21.29%	25.16%	38.63%	3.93%	34.70%	3.87%
Qantas Airways	18.57%	20.28%	31.05%	3.16%	27.89%	1.71%
Westpac Banking Corporation	12.53%	13.29%	24.30%	2.47%	21.83%	0.76%
Average	15.45%	16.99%	27.15%	2.76%	24.39%	1.54%

Note: Columns (1) and (2) show the standard put and reset put premiums as a percentage of the underlying stock price using the valuation formulas (8) and (7). Column (3) shows the put premium implied from quoted GEI interest rates. Column (4) provides the maximum tax benefit as a result of interest prepayment (assuming a 48.5% marginal tax rate), and column (5) shows the put premium of the GEI after tax relative to a standard ASX put option. Column (6) contains the reset premium—the difference between the reset put value and the standard put value. All calculations are based on market interest rates, implied volatilities from at-the-money options trading on the ASX, and historical dividend yields.

In Table 4, the *reset premium* in the sixth column is expressed as the difference between the value of a European-style put option with a reset feature (7) and the value of an otherwise identical put without the reset feature (8). Once again, all figures are expressed as a percentage of the underlying stock price. Naturally, the reset put premium should be higher than the standard put. Indeed, Table 4 shows that the average reset put premium is 16.99%, more than 1.54% higher than the average premium for the standard put. Interestingly, Macquarie Bank currently quotes a standard reset premium of 0.25% p.a. for a single reset, regardless of the investor's choice of reset date. This equates to a total reset premium of 0.72%[5] for all stocks. All of the reset premiums in the final column of Table 4 are greater than 0.72%. In other words, Macquarie Bank appears to overprice the value of the insurance embedded in the GEI (i.e. charges an average of 27.15% with an average cost of 15.45%) and underprice the value of the reset feature (i.e. charges an average of 0.72% with an average cost of 1.54%).

Table 5

Implied Volatilities from ASX Options and Macquarie Geared Equity Investments on 3 June 1998

	(1)	(2)	(3)	(4)
	ASX Options Implied Volatility	One-Year GEI: No Reset Implied Volatility	Three-Year GEI: No Reset Implied Volatility	Three-Year GEI: One Reset Implied Volatility
Amcor Limited	23.07%	30.92%	59.54%	32.17%
ANZ Banking Group	30.07%	34.37%	67.73%	36.69%
Broken Hill Proprietary Company Ltd	26.06%	31.89%	61.41%	34.26%
Brambles Industries	23.95%	32.29%	57.90%	33.64%
Boral Limited	29.70%	29.51%	58.02%	31.90%
Commonwealth Bank of Australia	20.88%	31.04%	61.51%	32.89%
Coca-Cola Amatil	39.10%	50.21%	79.34%	43.86%
Coles Myer Limited	24.90%	32.41%	54.69%	31.41%
CSR Limited	30.72%	32.54%	61.77%	33.70%
Foster's Brewing Group Ltd	24.30%	32.61%	59.37%	33.65%
Lend Lease Corporation Ltd	28.49%	40.51%	75.04%	41.11%
MIM Holdings Ltd	43.21%	38.69%	67.13%	37.31%
National Australia Bank	22.19%	33.45%	62.60%	34.25%
The News Corporation Ltd	26.34%	31.15%	55.17%	33.58%
Oil Search Limited	43.91%	53.49%	97.20%	52.48%
Qantas Airways	31.21%	34.67%	73.22%	38.71%
Westpac Banking Corporation	22.95%	30.00%	57.57%	32.14%
Average	28.89%	35.28%	65.25%	36.10%

5. The present value of three instalments of 0.25% is 0.72%.

Note: Column (1) shows the implied volatility from the longest-dated options trading on the ASX. These options have maturities between six and twelve months. Columns (2) through (4) show implied volatilities from various GEI options. In each case, the implied volatility is computed from the price of the GEI net of the potential tax benefit. All calculations are based on market interest rates and historical dividend yields.

The assessments of mispricing may be sensitive to our proxy of expected future stock return volatility. Recall that the option values reported in Tables 2 through 4 are based on implied volatilities of ASX options with times to expiration between six and twelve months, while the GEI securities valued range in maturity up to three years. To test the robustness of the reported results, implied volatilities from the prices of the GEI securities are computed and compared to the ASX option-implied volatilities.[6] The results are reported in Table 5. Our conclusions hold firm. Even for the implied volatilities of one-year GEI's reported in Column (2) of Table 5, which are the closest match to the maturities of the ASX options, the average implied volatility is 22% higher than for the ASX options. For the three-year GEI's in Column (3), the result is even more striking with the average GEI implied volatility being more than double the implied volatility from corresponding ASX options (65.25% versus 28.89%). Finally, the dramatic differences between the implied volatilities of the three-year GEI's without and with the reset feature in Columns (3) and (4) demonstrate clearly that the reset feature is being underpriced.

6. Summary and Conclusions

A reset put option is similar to a standard put option except that the exercise price is reset equal to the stock price on the pre-specified reset date if this stock price exceeds the original exercise price. In this paper we derive a valuation formula for a reset put option and present a range of comparative statics designed to highlight the differences between a reset put and a standard put. We also develop a numerical technique for valuing American-style reset puts. Finally, we apply our valuation results to assess the interest rate premiums embedded the Geared Equity Investments offered by Macquarie Bank and find that the premiums are, on average, almost 250 basis points higher than those of comparable ASX puts—even after considering the tax benefits available to GEI investors.

Our analysis of the reset put option valuation provides the groundwork for at least three other areas of investigation. First, how does the valuation equation change if the put option has two or more reset dates? For long-term put options, having the exercise price ratchet up as the stock price rises would seem very useful, particularly for portfolio insurers. Second, in the event the investor is allowed to choose reset dates, what is the optimal timing? Third, what if the reset dates are not set in advance? Here, valuation becomes more complicated but reduces to an optimal stopping problem. We are exploring these, and other, avenues in ongoing research.

(Date of receipt of final typescript: February 1999
Accepted by Garry Twite, Area Editor.)

6. We are grateful to the referee for suggesting this robustness test.

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